NEXT IAS

QUANTITATIVE APTITUDE

(Basic Numeracy & Data Interpretation)

Comprehensive Study Course

CIVIL SERVICES EXAMINATION 2026

Published by

MADE EASY PUBLICATIONS

MADE EASY Publications Pvt. Ltd.
Corporate Office: 44-A/4, Kalu Sarai
(Near Hauz Khas Metro Station), New Delhi-110016
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Quantitative Aptitude

First Edition: 2018
Second Edition: 2019
Third Edition: 2020
Fourth Edition: 2021
Fifth Edition: 2022
Sixth Edition: 2023
Seventh Edition: 2024

Eighth Edition: 2025

Contents

QUANTITATIVE APTITUDE

(Basic Numeracy & Data Interpretation)

UPSC SYLLABUS FOR CSAT

Total Marks : 200 **Duration : Two hours**

- Comprehension;
- Interpersonal skills including communication skills;
- Logical reasoning and analytical ability;
- Decision making and problem solving;
- General mental ability;
- Basic numeracy (numbers and their relations, orders of magnitude, etc.) (Class X level), Data interpretation (charts, graphs, tables, data sufficiency etc. — Class X level);

Paper-II of the Civil Services (Preliminary) Examination will be a qualifying paper with minimum qualifying marks fixed at 33%. The questions will be of multiple choice, objective type.

PREFACE

The journey to civil service examinations is one that is filled with dedication, perseverance, and relentless hard work. The Civil Services Aptitude Test (CSAT) is a crucial part of this journey, as it serves as the gateway to the prestigious Indian Civil Services. It is with great pleasure and immense pride that we present to you this book on "Quantative Aptitude" prepared by the NEXT IAS team under the guidance of **"Manjul Kumar Tiwari Sir"**.

The primary aim of this book is to provide aspirants with a thorough understanding of the CSAT examination pattern, the types of questions asked, and the best strategies to solve them. By providing detailed solutions to previous year questions, we hope to instill in you the confidence and ability to tackle any challenge that the CSAT may throw your way.

01 Chapter — Number System

Numerals

A mathematical symbol representing a number in a systematic manner is called a numeral. It is represented by a set of digits.

How to write a Number

To write a number, we put digits from right to left at the places designated as units, tens, hundreds, thousands, ten thousands, lakhs, ten lakhs, crores, ten crores and so on.
Let us see how the number 8765432
It is read as:

Ten Lakhs	Lakhs	Ten Thousands	Thousands	Hundreds	Tens	Unit
10^6 ↓	10^5 ↓	10^4 ↓	10^3 ↓	10^2 ↓	10^1 ↓	10^0 ↓
8	7	6	5	4	3	2

Number System

A system in which we study different types of numbers, as well as the relationship and rules that govern them is called a number system.
In the Hindu-Arabic system, we use the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These symbols are called digits.

Face Value of a Digit

In a numeral, the face value of a digit is the value of the digit itself irrespective of its place in the numeral.

Illus. In the numeral 38732, the face value of 8 is 8, the face value of 7 is 7, the face value of 2 is 2, the face value of 3 is 3, and so on.

Place Value (or local Value) of a Digit

In a numeral, the place value of a digit depends on its position in the number.

Illus. Look at the following to get the idea of place value of digits in 213764

Lakhs → Place value of 2 → 2×100000 = 200000

Ten Thousands → Place value of 1 → $1 \times 10000 = 10000$

Thousands → Place value of 3 → 3×1000 = 3000

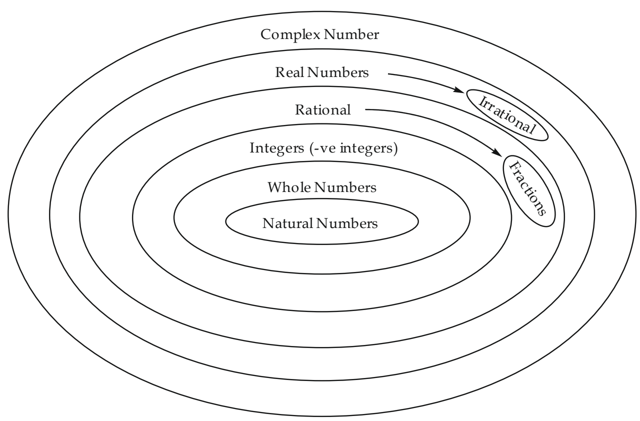
Hundreds → Place value of 7 → 7×100 = 700

Tens → Place value of 6 → $6 \times 10 = 60$

Units → Place value of 4 → $4 \times 1 = 4$

It is clear from the above presentation that to obtain the place value of a digit in a numeral, we multiply the digit with the value of its place in the given numeral.

Classifications of Numbers



Types of Numbers

Natural Numbers

- Counting Numbers 1, 2, 3, 4, ... upto infinity are called Natural Numbers
- These numbers start with '1'

Whole Numbers

- All the natural numbers together with '0' are called Whole Numbers

Integers

- An integer is a number that can be written without a fractional component
- Integers are further classified into :
  - Positive integers (1, 3, 4 etc.)
  - Zero (0)
  - Negative integers (-2, -4 etc.)

Even Numbers

- All whole numbers divisible by 2 are considered as even numbers. Hence 0, 2, 4 ... are even numbers
- The unit place of an even number will be 0, 2, 4, 6, 8 only

Note: [–2, –4, –6, ...] are even integers but they are not even numbers

Odd Numbers

- All whole numbers not divisible by 2 are odd
- Hence, 1, 3, 5, 7, ... are odd numbers
- The unit place of an odd number will be 1, 3, 5, 7 and 9 only

Note: [–1, –3, –5, ...] are odd integers. But they are not odd numbers

Properties of numbers based on Even and Odd

Even + Even = Even

Odd + Even = Odd

Odd + Odd = Even

Odd × Odd = Odd

Odd × Even = Even

Even × Even = Even

$(\text{Even})^{\text{Odd}} = \text{Even}$

$(\text{Odd})^{\text{Even}} = \text{Odd}$

$(\text{Even})^{\text{Even}} = \text{Even}$

$(\text{Odd})^{\text{Odd}} = \text{Odd}$

Ex. There are two, 2-digit numbers ab and cd, if $ab \times cd = 273$, what is the sum of $(ab + cd)$ = ?

(a) 21 (b) 35

(c) 47 (d) 66

Sol. (d)

$ab \times cd$ is odd

It means ab and cd both are odd

Hence their sum must be even, only one option is there which is even

Rational Numbers

- A number which can be expressed in the form of $\frac{p}{q}$ where, p and q are integers and $q \neq 0$, is called a rational number. Other examples of rational number are $\frac{-2}{5}, \frac{3}{4}$ etc.
- Rational Numbers have following forms of representations:

(i) Terminating decimal forms

Illus. $0.24 = \frac{24}{100}$ = Rational number

(ii) Nonterminating but recurring decimal forms

Illus. 0.121212 ...

Let A = 0.121212 ...

100A = 12.121212 ...

$100A - A = 99A = 12 \Rightarrow A = \frac{24}{99}$ = Rational number

Note: Any integer number is rational number since it can be written as the ratio of two integer numbers

Irrational Numbers

- A real number, which is not rational, is called an irrational number. An irrational number has non-terminating and non-recurring decimal part
- Between any two numbers, there are infinite numbers of irrational numbers
- Examples are : $\sqrt{3}, \sqrt{5}, \sqrt[3]{7}, \sqrt[4]{11}, \pi$ etc.

Note: Any terminating or recurring decimal is a rational number. Any non-terminating and non-recurring decimal is an irrational number

Real Numbers

- The real numbers include all the measuring numbers
- All the numbers which can be represented on the number line are called real numbers

Complex Numbers

- All the numbers that can be represented in $a + ib$ form where a and b real numbers and $i = \sqrt{-1}$ are called Complex Numbers

Ex. Which one of the following is not a rational number?

(a) $\frac{4}{7}$ (b) $\frac{-3}{11}$

(c) $\sqrt{3}$ (d) None of these

Sol. (c)

The numbers in option (a) and (b) are rational numbers, as they are the ratio of two integers. The number $\sqrt{3}$ is non-recurring, so it is not a rational number

Ex. The value of $1.\overline{34}+4.1\overline{2}$ is

(a) $\frac{37}{90}$ (b) $\frac{278}{90}$

(c) $3\frac{278}{99}$ (d) $5\frac{461}{990}$

Sol. **(d)**

$1.\overline{34}$ can be written as

$1.\overline{34}=1.343434.....$ ...(i)

As the bar is placed on two digits after decimal point, so, we will multiply the above equation by 10^2

$1.\overline{34}\times 10^2=134.3434.....$ ...(ii)

From equation (i)and (ii)

$(10^2-1)1.\overline{34}=133$

$\therefore \quad 1.\overline{34}=\frac{133}{99}$

In $4.1\overline{2}$, the bar is place on one digit so it can be written as $4.1\overline{2}=4.1222....$ ...(iii)

by multiplying with 10^2 in above equation,

$4.1\overline{2}\times 10^2=412.222....$ ...(iv)

by multiplying with 10, equation (i) can be written as $4.1\overline{2}\times 10=41.222....$ ...(v)

From equation (iv)and (v)

$4.1\overline{2}(100-10)=412-41=412-41$

$\therefore \quad 4.1\overline{2}=\frac{371}{90}$

Hence, the required value is

$1.\overline{34}+4.1\overline{2}=\frac{133}{99}+\frac{371}{90}$

$=\frac{1330+4081}{990}=\frac{5411}{990}=5\frac{461}{990}$

Ex. Consider the following statements:

(i) Every whole number is a real number

(ii) Every real number is a rational number

(iii) Every integer is a real number

(iv) Every rational number is a real number

Which of the above statements are correct?

(a) I and II (b) I, III and IV

(c) I, II and III (d) II and IV

Sol. (b)

We know that all whole numbers are real numbers but its converse is not true. So, statement (I) is true.

Every real number is not a rational number, some may be irrational numbers. Hence, statement (II) is wrong

Similarly, can say about statement (III) and (IV) that both the statements are true

Prime Numbers

- Prime numbers are those numbers, which do not have any factor apart from 1 (one) and itself
- Examples of prime numbers are : 2, 3, 5, 7, 11, 13, 17, 19 etc.
- A natural number is called a prime number, if it has exactly two factors, namely 1 and the number itself
- There are 25 prime numbers between 1 to 100
- 2 is the only even number which is prime
- A prime number is always greater than 1
- 1 is not a prime number. Therefore, the lowest odd prime number is 3
- Every prime number greater than 3 can be represented by $6n\pm1$, where n is integer. but the converse is not true

Steps to check whether a given number is prime or not

- Let the number be n
- Take the square root of n
- If it is a natural number, consider it as it is and if is not a natural number, increase the square root of it to the next natural number
- Now divide the given number by all the prime numbers that lies below the square root obtained
- If the given number is divisible by any of these prime numbers then it is not a prime number else it is a prime number

Illus. Check whether 352 is prime or not?

Sol. Square root of 352 lies between 18 and 19

So, we consider it as 19

Now prime numbers upto 19 are : 2, 3, 5, 7, 11, 13, 17 and 19

We will divide 352 by all the above prime numbers

$\because$ 352 is divisible by 11 so it is not a prime number

Relative Primes

- Two integers are relative primes or co-primes, if they share no common positive factors (divisors) except 1.

Illus. The numbers 4 and 9 do not have any common positive factors (s) hence, they are relative primes. But 4 and 6 are not relative primes.

Composite Numbers

- Numbers greater than 1 which are not prime, are known as composite numbers. They must have atleast one factor apart from 1 and itself.
- 1 is neither prime nor composite.
- Composite numbers can be both odd and even.

Illus. 4, 6, 8, 9, 10, 12 etc.

Co-prime Numbers

- Two natural numbers are said to be co-prime, if their HCF is 1. Any two consecutive natural numbers as well as any two prime numbers are always co-prime to each other.
- Co-prime numbers may or may not be prime.

Illus. (7, 8), (3, 5), (4, 9) etc.

Perfect Numbers

- If the sum of all the factors of a number, including the factor 1 and excluding the number itself, is equal to the number itself then, that number is called a perfect number.

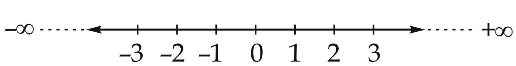
Illus. $28 = 1 \times 2 \times 2 \times 7 = 4 \times 7 = 2 \times 14$

Hence, the different factors are (1, 2, 4, 7, 14) and the sum of these is $1 + 2 + 4 + 7 + 14 = 28$.

Other perfect numbers are 6, 496, 8426 etc.

Number Line

Number line is a line on which all the positive and negative numbers can be represented in a sequence. It stretches from negative infinity to positive infinity.



Operations on Numbers

Let us understand some basic mathematical operations involving numbers

Addition

- When two or more numbers are combined together, then it is called addition.
- It is denoted by '+' sign

Illus. $38 + 43 + 19 = 100$

Subtraction

- When one or more numbers are taken out from another number, then it is called subtraction.
- Subtraction is denoted by '–' sign

Illus. $38 - 14 - 3 = 21$

Multiplication

- When 'a' is multiplied by 'b', then 'a' is added 'b' times or 'b' is added 'a' times. It is denoted by '×'

Illus. If a = 4 and b = 5, then $4 \times 5 = 20$ or $(4 + 4 + 4 + 4 + 4) = 20$

Here, 'a' is added 'b' times or in other words 4 is added 5 times

Similarly, $5 \times 4 = 20$ or $(5 + 5 + 5 + 5) = 20$

In this case, 'b' is added 'a' times or in other words 5 is added 4 times

Division

- When D and d are two numbers, then $\frac{D}{d}$ is called the operation of division, where D is the dividend and d is the divisor. A number which tells how many times a divisor (d) exists in dividend D is called the quotient Q
- If dividend D is not a multiple of divisor d, then D is not exactly divisible by d and in this case a remainder R is obtained

Illus. Let D = 21 and d = 4

Then, $\frac{D}{d} = \frac{21}{4} = 5\frac{1}{4}$

Here, 5 = Quotient (Q),

4 = Divisor (d),

and 1 = Remainder (R)

Dividend = Divisor × Quotient + Remainder

Fractions

- A fraction is a numerical quantity that is not a whole number
- It represents a part of a whole
- It is expressed in the form as $\frac{\text{numerator}}{\text{denominator}}$

Illus. Find 13% of 7

Sol. 13% of 7 = $\frac{13}{100}\times 7 = \frac{91}{100}$

Types of Fractions:

Fractions can be broadly classified in 3 types:

1. **Proper fractions :** Any fraction whose value < 1

   i.e., numerator < denominator

   e.g. $\frac{3}{5}, \frac{2}{7}, \frac{19}{23}$ etc.

2. **Improper fractions :** Any fraction whose value > 1

   ie. numerator > denominator

   e.g. $\frac{4}{3}, \frac{17}{11}, \frac{9}{5}$ etc.

3. **Mixed fractions:** It is another way of writing an improper fraction

   It consists of a natural number and a fraction. A mixed fraction is always greater than 1

   e.g. $2\frac{1}{3}, 4\frac{2}{7}, 3\frac{8}{11}$ etc.

Note: All the natural numbers can be written in the form of a fraction where denominator is always 1

Ex. Express $\frac{11}{3}$ in mixed fraction.

Sol. $\frac{11}{3} = \frac{9+2}{3} = \frac{9}{3}+\frac{2}{3} = 3+\frac{2}{3} = 3\frac{2}{3}$

Ex. Convert $12\frac{2}{7}$ into improper fraction.

Sol. $12\frac{2}{7} = 12+\frac{2}{7} = \frac{12\times 7+2}{7} = \frac{86}{7}$

Illus. Solve $\frac{2}{3}+\frac{5}{8}$

Sol. To solve such questions involving fractions, we follow the following steps:

1. Take LCM of denominators.
2. Divide the LCM with each denominator and multiply the quotient with corresponding numerators.
3. Add the resultant numerators.

$$\therefore \frac{2}{3}+\frac{5}{8} = \frac{2\times 8+5\times 3}{24} = \frac{31}{24}$$

Ex. Find the value of $\frac{3}{7}+\frac{4}{9}$

Sol. $\frac{3}{7}+\frac{4}{9} = \frac{9\times 3+4\times 7}{7\times 9} = \frac{27+28}{63} = \frac{55}{63}$

Ex. Solve : $\dfrac{5\times\frac{4}{8}+4\times\frac{3}{7}}{\frac{2}{7}\div\frac{4}{21}}$

Sol. $\dfrac{5\times\frac{4}{8}+\frac{4\times 3}{7}}{\frac{2}{7}\times\frac{21}{4}} = \dfrac{\frac{20}{8}+\frac{12}{7}}{\frac{3}{2}} = \left(\frac{20}{8}+\frac{12}{7}\right)\times\frac{2}{3}$

$$= \left(\frac{140+96}{56}\right)\times\frac{2}{3} = \frac{236}{56}\times\frac{2}{3} = \frac{236}{28\times 3} = \frac{59}{21}$$

Types of Decimal Fraction

1. **Recurring decimal Fraction:** The decimal fraction, in which one or more decimal digits are repeated again and again, is called recurring decimal fraction. To represent these fractions, a line is drawn on the digits which are repeated

Illus.

(a) $\frac{1}{3} = 0.3333 = 0.\overline{3}$,

(b) $\frac{22}{7} = 3.142857142857 = 3.\overline{142857}$

2. **Pure Recurring Decimal Fraction:** When all the digits in a decimal fraction are repeated after the decimal point, then the decimal fraction is called as pure recurring decimal fraction

   e.g. $0.\overline{5}, 0.\overline{489}$ etc.

   To convert pure recurring decimal fractions into simple fractions, write down the repeated digits only once in numerator and place as many nines, in the denominator as the number of digits repeating

Illus.

(a) $0.\overline{6} = \frac{6}{9} = \frac{2}{3}$

Sol. Since, there is only 1 repeated digit.
Therefore, only single 9 is placed in denominator.

(b) $0.\overline{36} = \frac{36}{99} = \frac{3}{11}$

Sol. Since, there are only 2 repeated digits.
Therefore, two 9's are placed in denominator.

3. Mixed Recurring Decimal Fraction: A decimal fraction in which some digits are repeated and some are not repeated after decimal is called as mixed recurring decimal fraction.

e.g. $4.1\overline{2}$, $0.1\overline{23}$ etc.

Illus. 35.5 + 23.2 + 43.23 = ?

Sol.
$$\begin{array}{r} 35.50 \\ 23.20 \\ +43.23 \\ \hline 101.93 \\ \hline \end{array}$$

Ex. 4.3 × 0.13 = ?

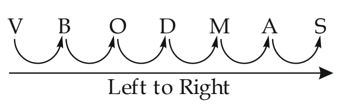
Sol. 43 × 13 = 559

Sum of the decimal places = (1 + 2) = 3

$\therefore$ Required product = 0.559

'VBODMAS' RULE

To simplify arithmetic expression involving various operations like brackets, multiplication, addition etc., a particular sequence of the operations is followed
The operations have to be carried out in the order, in which they appear in the word stand for following operations
Order of above mentioned operations is same as the order of letters in the 'VBODMAS' from left to right as



First: Vinculum (*V*) or Bar '$\overline{\quad\quad}$'

Illus: –7 – 11 = – 18, but $-\overline{7-11} = -(-4) = 4$

Second: Brackets (*B*)

Order of removing brackets

1 Small Brackets (Circular brackets) '( )'

2. Middle brackets (Curly brackets) '{ }'

3. Square brackets (Big Brackets) '[ ]'

Third: Operation of 'Of' (*O*)

Fourth: Operation of division — (*D*)

Fifth: Operation of multiplication — (*M*)

Sixth: Operation of addition — (*A*)

Seventh: Operation of subtraction — (*S*)

Ex. Solve: $3+8\times(70\div28)+8$ of $\overline{3-1}$

Sol. Following VBODMAS Rule,
Expression = 3 + 8 × (70 ÷ 28) + 8 of 2
= 3 + 8 × (70 ÷ 28) + 16

$= 3+8\times\left(\frac{5}{2}\right)+8\times2$

= 3 + 20 + 16 = 39

Ex. Simplify the following expression:

$$\frac{5\times1.6+4\div2-8}{10+6\times2-4-\overline{3\times8}\div2}$$

Sol. Following VBODMAS Rule,

Expression = $\frac{5\times1.6+2-8}{10+6\times2-4-24\div2}$

$= \frac{8+2-8}{10+6\times2-4-12}$

$= \frac{10-8}{10+12-4-12} = \frac{2}{22-16} = \frac{2}{6} = \frac{1}{3}$

Some Important Algebraic Formulae:

1. $(a + b)^2 = a^2 + 2ab + b^2$

2. $(a - b)^2 = a^2 - 2ab + b^2$

3. $(a - b)(a + b) = a^2 - b^2$

4. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

5. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

6. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

7. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Solved Examples

Q.1 If x is a positive integer and $(x + 3)(x + 5)$ is odd, then $x(x + 2)$ must be a multiple of which one of the following?

(a) 3 (b) 6

(c) 8 (d) 12

Sol. **(c)**

$(x + 3)(x + 5)$ is odd only when both $(x + 3)$ and $(x + 5)$ are odd. This is possible only when x is even. Hence, $x = 2y$, where y is a positive integer. Then,

$x(x + 2) = 2y(2y + 2) = 2(y)2(y + 1) = 4(y)(y + 1) = 4$ (product of two consecutive positive integers, out of which one must be even) = 4 (an even numbers), and this equals a number that is at least a multiple of 8.

Q.2 6 is added to a certain number and the sum is multiplied by 4; the product is divided by 18 and 3 is subtracted from the quotient. The remainder left is 1. The number is:

(a) 10 (b) 12

(c) 30 (d) 18

Sol. **(b)**

Let the number be x.

Then,

$\Rightarrow \dfrac{4(x+6)}{18} - 3 = 1$

$\Rightarrow \dfrac{4(x+6)}{18} = 4$

$\Rightarrow x + 6 = 18$

$\Rightarrow$ x = 12

Q.3 A class starts at 10 a.m. and lasts till 1 : 26 p.m. Three periods are held during this interval. After every period, 10 minutes are given free to the students. The exact duration of each period is:

(a) 48 minutes (b) 51 minutes

(c) 56 minutes (d) 62 minutes

Sol. **(d)**

Time between 10 a.m. and 1 : 26 p.m. = 3 hrs. 26 min. = 3 × 60 + 26 = 206 min

For two periods in between, free time = 2 × 10 = 20 min

Remaining time = (206 – 20) min = 186 min

Duration of each of the 3 periods = $\dfrac{186}{3}$ min

= 62 minutes

Q.4 A boy wrote all the numbers from 200 to 300. Then he started counting the number of two's that have been used while writing all these numbers. What is the number that he got?

(a) 110 (b) 115

(c) 120 (d) 130

Sol. **(c)**

From 200 to 300 there are 101 numbers

There are 100 2's in the hundred place

10 2's in tens place, 10 2's in unit place

Thus number of 2's = 100 + 10 + 10 = 120

Q.5 If each of a, b, c are divisible by 3 $(a, b, c \neq 0)$ then abc must be divisible by which one of the following?

(a) 3 (b) 9

(c) 18 (d) None of these

Sol. **(d)**

Since each one of the three numbers a, b and c is divisible by 3.

Let $a = 3x$, $b = 3y$, $c = 3z$, where x, y, z are non-zero integers.

$\Rightarrow abc = 27\, xyz$

Since x, y and z are integers, xyz is an integer and therefore, abc is divisible by 27.

Q.6 The difference between the squares of two consecutive even integers is always divisible by:

(a) 3 (b) 5

(c) 4 (d) 8

Sol. **(c)**

Let the two consecutive even integers be $2x$ and $2x + 2$

Then, $(2x + 2)^2 – (2x)^2 = (2x + 2 + 2x)(2x + 2 – 2x)$ $= (4x + 2)(2)$

$= 4(2x + 1)$ which is always divisible by 4

Q.7 The smallest value of n, for which $6n + 1$ is not a prime number, is

(a) 1 (b) 2

(c) 3 (d) 4

Sol. **(d)**

Putting values for n,

$n = 1, 6n + 1 = 7$

$n = 2, 6n + 1 = 13$

$n = 3, 6n + 1 = 19$

$n = 4, 6n + 1 = 25$ (not a prime)

Q.8 The product of three consecutive natural numbers, the first of which is an odd number, is always divisible by

(a) 12 (b) 24

(c) 8 (d) 6

Sol. (d)

Let three consecutive numbers be $(2n+1)(2n+2)(2n+3)$. This will always be divisible by 6.

Q.9 If a and b are integers and $a - b$ = even , then which of the following is always even ?

(a) ab (b) $a^2 + b^2$

(c) $a^2 + b^2 + 1$ (d) None of these

Sol. (b)

$a - b$ = even

$\Rightarrow$ Both a , b are either odd or even

Let, $a = 1$, $b = 3$

ab = odd

$a^2 + b^2$ = even

$a^2 + b^2 + 1$ = odd

Let $a = 2$, $b = 4$

ab = even

$a^2 + b^2$ = even

$a^2 + b^2 + 1$ = odd

$a^2 + b^2$ = odd

$\Rightarrow a^2 + b^2$ is always even

Previous Years Solved Questions

Q.1 A club has 108 members. Two thirds of them are men and the rest are women. All members are married except for 9 women members. How many married women are there in the club?

(a) 20 (b) 24

(c) 27 (d) 30

Sol. (c)

Given,

Total member = 108

Female member = $108 \times \frac{1}{3} = 36$

Given, 9 women are unmarried.

So, number of married women = 36 – 9 = 27

Q.2 If all numbers from 501 to 700 are written. What is the total number of times does the digit 6 appears?

(a) 138 (b) 139

(c) 140 (d) 141

Sol. (c)

For numbers between 600 to 700:

Number of 6 at the units place = 10

Number of 6 at the tens places = 10

Number of 6 at hundredth place = 100

For number between 501 to 599:

Number of 6 at the units place = 10

Number of 6 at the tens places = 10

Hence, total number of 6 between (501 – 700)

= 10 + 10 + 100 + 10 +10 = 140

Q.3 A person is standing on the first step from the bottom of a ladder. If he has to climb 4 more steps to reach the middle step, low many steps does the ladder have.

(a) 8 (b) 9

(c) 10 (d) 11

[UPSC-2016]

Sol. (b)

As per the question:

- he is on the first step
- need four step to get to the middle rung of the ladder.
- Thus the 1 + 4 = 5th rung/step is the middle of the ladder.

It means there would be 9 steps in the ladder.

Options (b) is correct.

Q.4 X and Y are natural numbers other than 1, and Y is greater than X. Which of the following represents the largest number?

(a) XY (b) X / Y

(c) Y / X (d) (X + Y) / XY

[UPSC-2018]

Sol. (a)

Y > X > 1

X/Y > X/Y as X/Y < 1

XY > Y/X as Y/X < Y and XY > Y

$XY > \frac{X+Y}{XY}$ as $XY > \frac{X}{Y} + \frac{1}{X}$ as $\frac{1}{Y} + \frac{1}{X} < 2$

but XY > 2.

Q.5 If $x - y = 8$, then which of the following must be true?

1. Both x and y must be positive for any value of x and y.
2. If x is positive, y must be negative for any value of x and y.
3. If x is negative, y must be positive for any value of x and y.

Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2 nor 3

[UPSC-2018]

Sol. (d)

$x = 10$ $y = 2$
$x = -1$ $y = -9$
$y = -3$ $x = 5$

If $x > 0$, y can be both > 0 or < 0

If $x < 0$, $y < 0$

Q.6 The number of times the digit 5 will appear while writing the integers from 1 to 1000 is

(a) 269 (b) 271
(c) 300 (d) 302

[UPSC-2019]

Sol. (c)

Q.7 Let p, q, r and s be natural numbers such that p –2016 = q + 2017 = r – 2018 = s + 2019 Which one of the following is the largest natural number?

(a) p (b) q
(c) r (d) s

[UPSC-2020]

Sol. (c)

$p - 2016 = q + 2017$

$p = q + 2017 + 2016$

Similarly,

$r = q + 2017 + 2018$

$r = s + 2018 + 2019$

Clearly, $r > s$, $r > q$

Also, $r = p + 2$

Therefore, r is largest

Hence (c)

Q.8 Let A3BC and DE2F be four-digit numbers where each letter represents a different digit greater than 3. If the sum of the numbers is 15902, then what is the difference between the values of A and D ?

(a) 1 (b) 2
(c) 3 (d) 4

[UPSC-2020]

Sol. (c)

Since A, B, C, D, E, F are all > 3

C + F = 12

B + 2 = 9, B =7

E + 3 = 8

E = 5

A + D = 15

Since all variables are distinct and > 3

A and D can only be 6 and 9

Difference of A and D = 3

Hence (c)

Q.9 How many pairs of natural numbers are there such that the difference of whose squares is 63?

(a) 3 (b) 4
(c) 5 (d) 2

[UPSC-2020]

Sol. (a)

Let the required pair of natural numbes is x and y

ATQ, $x^2 - y^2 = 63$

or $(x + y)(x - y) = 63$

There are three possible cases in which product of two numbers is 63.

Case 1: $(x + y) = 9$ and $(x - y) = 7$

Then x = 8 and y = 1

Case 2: $(x + y) = 21$ and $(x - y) = 3$

Then $x = 12$ and $y = 9$

Case 3: $(x + y) = 63$ and $(x - y) = 1$

Then $x = 32$ and $y = 31$

Hence, there are three pairs of natural numbes such that the difference of their squares is 63.

Q.10 Integers are listed from 700 to 1000. In how many integers is the sum of the digits 10?

(a) 6 (b) 7
(c) 8 (d) 9

[UPSC-2021]

Sol. (d)

Numbers whose sum of digits is 10 are:

703, 712, 721, 730, 802, 811, 820, 901 and 910

Hence, there are 9 such integers in which the sum of the digits is 10.

Q.11 Consider the following statements in respect of two natural numbers p and q such that p is a prime number and q is a composite number:

1. p × q can be an odd number
2. $\frac{q}{p}$ can be a prime number
3. p + q can be a prime number

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

[UPSC-2022]

Sol. (d)

p is a prime number. So, p can be 2, 3, 5, 7, 11, 13, ...

q is a composite number. So, q can be 4, 6, 8, 9, 10, ...

Statement 1: p × q can be an odd number, e.g. (3 × 9 = 27). Thus, statement 1 is correct.

Statement 2: $\frac{q}{p}$ can be a prime number, e.g. ($\frac{4}{2}$ = 2). Thus, statement 2 is correct.

Statement 3: p + q can be a prime number, e.g. (3 + 4 = 7)

Thus, statement 3 is correct.

Thus, all the statements 1, 2 and 3 are correct

Q.12 What is the remainder when 85 × 87 × 89 × 91 × 95 × 96 is divided by 100?

(a) 0 (b) 1

(c) 2 (d) 4

[UPSC-2023]

Sol. (a)

Remainder Calculation

The factorization of 100 is 4 x 5^2.

If we look at the given numbers, we can see that 85, 95 each contain a factor of 5, and 96 contain a factor of 4.

Therefore, we can say that the product of these numbers contains at least 4 x 5^2 = 100 as a factor.

This means that when the product of these numbers is divided by 100, the remainder will be 0.

So, the correct answer is (a).

Q.13 What is the unit digit in the expansion of $(57242)^{9\times7\times5\times3\times1}$?

(a) 2 (b) 4

(c) 6 (d) 8

[UPSC-2023]

Sol. (a)

Unit Digit of a Power $(5724)^{2945}$

To find the unit digit of $(5724)^{2945}$, we can focus solely on the unit digit of the base number, which is 2, since the unit digit of a power depends only on the unit digit of the base.

Calculating Power

First, calculate the product 9 x 7 x 5 x 3 x 1 = 945.

Pattern in Powers of 2

Next, observe the pattern in the powers of 2:

2^1 = 2 (unit digit is 2)

2^2 = 4 (unit digit is 4)

2^3 = 8 (unit digit is 8)

2^4 = 16 (unit digit is 6)

2^5 = 32 (unit digit is 2)

2^6 = 64 (unit digit is 4)

2^7 = 128 (unit digit is 8)

2^8 = 256 (unit digit is 6)

2^9 = 512 (unit digit is 2)

The unit digits repeat every four powers (2, 4, 8, 6). This is known as the cyclicity of 2, which is 4.

Finding Relevant Power

To find the relevant power in the cycle for 2945, calculate the remainder of 945 divided by 4:

945 ÷ 4 = 236 remainder 1

This remainder tells us that 2945 corresponds to the first number in the cycle of unit digits of 2, which is 2.

Thus, the unit digit of $(5724)^{2945}$ is 2.

Hence, option (a) is correct.

Q.14 D is a 3-digit number such that the ratio of the number to the sum of its digits is least. What is the difference between the digit at the hundred's place and the digit at the unit's place of D?

(a) 0 (b) 7

(c) 8 (d) 9

[UPSC-2023]

Sol. (c)

D is a 3-digit number

To minimize the ratio of a number to the sum of its digits, we need to maximize the sum of the digits and minimize the number itself.

The smallest 3-digit number is 100, but this doesn't maximize the sum of the digits.

The smallest 3-digit number with the maximum digit sum is 199 (1+9+9=19). This gives us a ratio of 10.47, which is the smallest possible for a 3-digit number.

The difference between the digit at the hundred's place (1) and the digit at the unit's place (9) of D is 9-1 = 8.

So, the correct answer is (c).

Q.15 Three of the five positive integers p, q, r, s, t are even and two of them are odd (not necessarily in order). Consider the following:

1. p + q + r - s - t is definitely even.
2. 2p + q + 2r - 2s + t is definitely odd.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

[UPSC-2023]

Sol. (a)

Odd-even integers p, q, r, s, t

The sum or difference of two even numbers is always even, and the sum or difference of two odd numbers is always even. So, the sum or difference of any number of even numbers is always even.

For statement 1, since p, q, r, s, t are five integers where three are even and two are odd, we can group the even and odd numbers separately. The sum of three even numbers minus the sum of two odd numbers is even – even, which is definitely even.

For statement 2, the expression 2p + q + 2r – 2s + t can be rearranged as 2(p + r – s) + (q + t). The term 2(p + r – s) is definitely even as it is a product of 2 and some integer. However, (q + t) is the sum of two integers where one is even and the other is odd, which is definitely odd. Therefore, the sum of an even number and an odd number is odd. So, statement 2 is not definitely odd, it depends on the values of q and t.

Therefore, only statement 1 is correct.

Q.16 Consider the following in respect of prime number *p* and composite number *c*.

1. $\frac{p+c}{p-c}$ can be even.
2. $2p + c$ can be odd.
3. pc can be odd.

Which of the statements given above are correct?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

[UPSC-2023]

Sol. (d)

Prime and Composite Number Analysis

To determine which statements are correct, let's analyze each one individually:

1. p + c / p – c can be even:

For this expression to be even, the numerator (p + c) and the denominator (p – c) must both be either even or odd.

Prime numbers (p) are mostly odd, except for 2.

Composite numbers (c) can be either even or odd.

If p is odd and c is even, then:

p + c is odd + even = odd

p – c is odd – even = odd

An odd number divided by an odd number can be even if the result is an integer.

If p is 2 (even) and c is odd, then:

p + c is even + odd = odd

p – c is even – odd = odd

An odd number divided by an odd number can be even if the result is an integer.

Therefore, it is possible for p + c / p – c to be even.

2. 2p + c can be odd:

For 2p + c to be odd:

2p is always even because 2 times any integer is even.

For the sum to be odd, c must be odd (even + odd = odd).

Since composite numbers can be odd, this statement is true.

3. pc can be odd:

For the product pc to be odd:

Both p and c must be odd.

Since prime numbers (except 2) are mostly odd and composite numbers can be odd, this statement is true.

Given the analysis, all three statements are correct. Hence, option (d) is correct.

Q.17 A 3-digit number ABC, on multiplication with D gives 37DD where A, B, C and D are different non-zero digits. What is the value of A + B + C ?

(a) 18
(b) 16
(c) 15
(d) Cannot be determined

[UPSC-2023]

Sol. (a)

3-digit number ABC

The problem states that when a 3-digit number ABC is multiplied by D, it gives 37DD. Here, A, B, C, and D are different non-zero digits.

To solve this, we need to find a value for D such that 37DD is a 4-digit number. The only possible values for D are 1, 2, 3, 4, 5, 6, 7, 8, and 9.

If D = 1, then 37DD is not a 4-digit number. If D = 2, then 37DD is not a 4-digit number. If D = 3, then 37DD is not a 4-digit number.

If D = 4, then 37DD = 3744, which is a 4-digit number. Therefore, D = 4.

Now, we need to find a 3-digit number ABC such that ABC * 4 = 3744.

The only 3-digit number that satisfies this equation is 936. Therefore, A = 9, B = 3, and C = 6.

Finally, we need to find the value of A + B + C.

A + B + C = 9 + 3 + 6 = 18.

Therefore, the correct answer is (a).

Q.18 For any choices of values of X, Y and Z, the 6-digit number of the form XYZXYZ is divisible by:

(a) 7 and 11 only (b) 11 and 13 only
(c) 7 and 13 only (d) 7, 11 and 13

[UPSC-2023]

Sol. (d)

Number XYZXYZ Analysis

The number of the form XYZXYZ can be expressed as follows:

XYZXYZ = XYZ * 1000 + XYZ

Now, factor out 1000 from the first term:

XYZXYZ = XYZ * (1000 + 1)

Simplify the expression inside the parentheses:

XYZXYZ = XYZ * 1001

So, the number of the form XYZXYZ is equal to XYZ * 1001.

The prime factorization of 1001 is 7 * 11 * 13. Therefore, any number of the form XYZ * 1001 will be divisible by 7, 11, and 13.

So, the correct answer is (d)

7, 11, and 13. You can verify this by dividing 111111 by 7, 11, or 13 and getting a whole number as the quotient.

Q.19 Let x be a positive integer such that $7x + 96$ is divisible by x. How many values of x are possible?

(a) 10 (b) 11
(c) 12 (d) Infinitely many

[UPSC-2023]

Sol. (c)

7x + 96 is divisible by x

The statement "7x + 96 is divisible by x" implies that when you divide the expression "7x + 96" by "x", you get a whole number (an integer) as a result. This means that "x" must be a factor of the constant term "96", because the "7x" term will always be divisible by "x" (since it contains "x" as a factor).

To find all the possible values of "x", we need to list all the factors of 96. The prime factorization of 96 is indeed (2^5 * 3). From this prime factorization, we can determine all the factors of 96 by taking various combinations of these prime factors.

Here are all the factors of 96:

1,2,4,8,16,32,3,6,12,24,48,96

These are the 12 factors of 96, which means that "x" can be any of these 12 values. If "x" is any of these values, then "7x + 96" will be divisible by "x". Therefore, the statement is correct, and there are indeed 12 possible values for "x".

Q.20 If p, q, r and s are distinct single digit positive numbers, then what is the greatest value of (p + q) (r + s)?

(a) 230 (b) 225
(b) 224 (d) 221

[UPSC-2023]

Sol. (b)

Greatest Value of Expression (p + q)(r+s)

The greatest value of (p + q)(r + s) will be achieved when p, q, r, and s are the largest distinct single digit positive numbers.

The largest distinct single digit positive numbers are 9, 8, 7 and 6.

So, the greatest value of (p + q)(r + s) is (9 + 8)(7 + 6) = 17 * 13 = 221.

However, if we rearrange the numbers as (9 + 7)(8 + 6), we get a larger value: 16 * 14 = 224.

But, the largest value is obtained when we rearrange the numbers as (9 + 6)(8 + 7) = 15 * 15 = 225.

So, the correct answer is (b).

Q.21 A number N is formed by writing 9 for 99 times. What is the remainder if N is divided by 13?

(a) 11 (b) 9
(c) 7 (d) 1

[UPSC-2023]

Sol. (a)

Number *n* is formed by writing 9 for 99 times

Group the given number into sets of 3 starting from the right, or the units place. From the rightmost group of 3 digits apply the subtraction and addition operations alternatively and find the result. If the result is either a 0 or it can be divided by 13 completely without leaving a remainder, then the number is divisible by 13.

For example, in the number 1,139,502 applying the subtraction and addition operations alternatively from the rightmost group of 3 digits, we get 502 – 139 + 1 = 364. 364/13 gives 28 as quotient and 0 as remainder. Therefore, 1139502 is divisible by 13.

Similarly,

To determine if the number formed by writing the digit 9 ninety-nine times is divisible by 13

We can use a method that involves grouping the digits and applying alternating addition and subtraction operations. Here's a step-by-step explanation:

1. Form the Number

The number in question is composed of the digit 9 repeated 99 times.

2. Group the Digits

We group the digits into sets of three, starting from the rightmost digit. This results in 33 groups of '999'.

3. Apply Operations

Starting from the rightmost group, we apply alternating subtraction and addition operations to these groups. This method is based on a divisibility rule for 13, which states that if you take the last three digits of a number, subtract the next three digits, add the next three, and so on, and if the resulting number is divisible by 13, then the original number is also divisible by 13.

4. Calculate

For our number, the calculation would look like this:

999 – 999 + 999 – 999 + … + 999 (33 times in total).

5. Simplify the Calculation

Since each '999' is either added or subtracted, and there are an odd number of these groups (33), the calculation simplifies to just one '999' (because 32 of them will cancel each other out).

6. Check Divisibility

Now, we check if 999 is divisible by 13:

999 ÷ 13 = 76 remainder 11.

Since there is a remainder when 999 is divided by 13, the original number (composed of 99 nines) is not divisible by 13.

Q.22 What is the sum of all digits which appear in all the integers from 10 to 100?

(a) 855 (b) 856
(c) 910 (d) 911

[UPSC-2023]

Sol. (b)

Sum of Digits of Intergers

Sum of the tens digits:

From 10 to 19, the tens digit is 1.

From 20 to 29, the tens digit is 2.

From 30 to 39, the tens digit is 3.

…

From 90 to 99, the tens digit is 9.

Each group contains 10 numbers.

Therefore, the sum of the tens digits is:

(1+2+3+4+5+6+7+8+9) x 10 = 45 x 10 = 450

Sum of the units digits:

The units digits from 10 to 100 repeat every ten numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

This sequence repeats 9 times (from 10 to 19, 20 to 29, …, 90 to 99).

Therefore, the sum of the units digits is:

(0+1+2+3+4+5+6+7+8+9) x 9 = 45 x 9 = 405

Adding the sums of tens and units digits:

450 + 405 = 855

However, we must also consider the number 100:

The tens digit is 10, and the units digit is 0.

The sum of digits in 100 is 1 + 0 + 0 = 1.

Adding this to our previous total:

855 + 1 = 856

Thus, the sum of all digits which appear in all the integers from 10 to 100 is 856. The correct answer is (b) 856.

Q.23 How many natural numbers are there which give a remainder of 31 when 1186 is divided by these natural?

(a) 6 (b) 7

(c) 8 (d) 9

[UPSC-2023]

Sol. (d)

Remainder is 31 when 1186 is divided by a certain number.

However, since the remainder is 31, the divisor must be greater than 31. So, the possible numbers are 33, 35, 55, 77, 105, 165, 231, 385, 1155. Hence, there are 9 such natural numbers.

Q.24 Let pp, qq and rr be 2-digit numbers where p < q < r. If pp + qq + rr = tt,where tt is a 3-digit number ending with zero,consider the following statements:

1. The number of possible values of p is 5.
2. The number of possible values of q is 6.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1and 2 (d) Neither 1nor 2

[UPSC-2023]

Sol. (c)

The problem is based on the concept of number theory and algebra.

The given condition is that pp, qq and rr are 2-digit numbers where p< q< r and their sum is a 3-digit number ending with zero.

This means that the sum of the digits p, q and r is either 10 or 20, as the sum of their double-digit representations (pp, qq, rr) is a three-digit number ending in zero (either 110 or 220).

For the first statement, we need to find the possible values of p. Since p is the smallest digit and the sum of p, q and r is either 10 or 20, the possible values of p can be 1 or 2 for the sum 10, and 1, 2, 3, 4 or 5 for the sum 20. So, there are 5 possible values for p, making the first statement correct.

For the second statement, we need to find the possible values of q. Since q is the middle digit and the sum of p, q and r is either 10 or 20, the possible values of q can be 2, 3 or 4 for the sum 10, and 6, 7 or 8 for the sum 20. So, there are 6 possible values for q, making the second statement correct.

Therefore, both statements are correct, and the answer is (c) Both 1 and 2.

Q.25 There are three traffic signals. Each signal changes colour from green to red and then from red to green. The first signal takes 25 seconds, the second signal takes 39 seconds and the third signal takes 60 seconds to change the colour from green to red. The durations for green and red colours are same. At 2:00 p.m, they together turn green. At what time will they change to green next, simultaneously?

(a) 4:00 p.m (b) 4:10 p.m

(c) 4:20 p.m (d) 4:30 p.m

[UPSC-2023]

Sol. (b)

The time taken by each signal to complete a full cycle (green to red and back to green) is twice the time taken to change from green to red. So, the first signal takes 50 seconds, the second signal takes 78 seconds, and the third signal takes 120 seconds to complete a full cycle.

To find out when all three signals will be green simultaneously again, we need to find the least common multiple (LCM) of these three times.

The LCM of 50, 78, and 120 is 7800 seconds.

7800 seconds is equivalent to 2 hours and 10 minutes.

So, if all three signals were green at 2:00 p.m., they will all be green simultaneously again at 4:10 p.m.

Therefore, the correct answer is (b) 4:10 p.m.

Q.26 What is the remainder if 2^{192} is divided by 6?

(a) 0 (b) 1
(c) 2 (d) 4

[UPSC-2023]

Sol. (d)

The powers of 2 cycle in a pattern of 4 when divided by 6.

Specifically, the remainders are 2, 4, 2, 0, 2, 4, 2, 0, and so on. This pattern repeats every 4 powers.

Since 192 is a multiple of 4, the remainder when 2^192 is divided by 6 is the same as the remainder when 2^4 is divided by 6, which is 4.

So, the answer is (d) 4.

Q.27 AB and CD are 2-digit numbers. Multiplying AB with CD results in a 3-digit number DEF. Adding DEF to another 3-digit number GHI results in 975. Further A, B, C, D. E, F, G, H, I are distinct digits. If E = 0, F = 8, then what is A+B+C equal to?

(a) 6 (b) 7
(c) 8 (d) 9

[UPSC-2023]

Sol. (a)

Since E = 0 and F = 8, the product of AB and CD must be a number in the form of 08X, where X is a digit from 1 to 9.

The only way to get a product of 08X is if one of the numbers (AB or CD) is a multiple of 4 and the other number ends with 2 or 7.

The possible pairs of numbers are (12, 34), (17, 24), (24, 17), (34, 12).

However, since the sum of DEF and GHI is 975, the sum of the digits D, G, H, I must be 9 (since E = 0 and F = 8).

The only pair of numbers that satisfies this condition is (12, 34), which gives a product of 408.

Therefore, A = 1, B = 2, C = 3, D = 4.

So, A + B + C = 1 + 2 + 3 = 6.

Therefore, the correct answer is (a) 6.

Q.28 What is the sum of all 4-digit numbers less than 2000 formed by the digits 1, 2, 3 and 4, where none of the digits is repeated?

(a) 7998 (b) 8028
(c) 8878 (d) 9238

[UPSC-2023]

Sol. (a)

As directed by the question, thousand place digit will always be 1, where 2, 3, 4 can be arranged in 3! ways giving six possible numbers.

4 digit numbers less than 2000 formed by digits 1, 2, 3, 4 where digits do not repeat are:

  1234
  1243
  1432
  1423
  1324
+ 1342
  7998

Hence, option (a) is correct.

Q.29 How many consecutive zeros are there at the end of the integer obtained in the product $1^2 \times 2^4 \times 3^6 \times 4^8 \times \ldots \times 25^{50}$?

(a) 50 (b) 55
(c) 100 (d) 200

[UPSC-2024]

Sol. (d)

We get a zero when we get 10, which in turn is obtained when 2 and 5 are multiplied. The number of zeroes will depend on whichever is less in number from among 2 and 5.

In the given product $1^2 \times 2^4 \times 3^6 \times 4^8 \times .... \times 25^{50}$, we can deduce that 2 is definitely more in number than 5. So, the number of zeroes at the end will be equal to the number of 5's in the given product.

The number of 5's in the given expression are:

There are 10 in 5^{10}

There are 20 in 10^{20}

There are 30 in 15^{30}

There are 40 in 20^{40}

There are 100 in 25^{50}

So, the number of zeroes at the end of the product = 10 + 20 + 30 + 40 + 100 = 200

Q.30 $222^{333} + 333^{222}$ is divisible by which of the following numbers?

(a) 2 and 3 but not 37

(b) 3 and 37 but not 2

(c) 2 and 37 but not 3

(d) 2, 3 and 37

[UPSC-2024]

Sol. (b)

$222^{333} + 333^{222}$

$= [(222)^3]^{111} + [(333)^2]^{111}$

$= [(2^3 \times 111^3)]^{111} + [(3^2 \times 111^2)]^{111}$

$= [(8 \times 111^3)^{111} + [(9 \times 111^2)]^{111}$

$= (111^2)^{111} \times [\{(8 \times 111)^{111} + (9)^{111}\}]$

$= 111^{222} \times [888^{111} + 9^{111}]$

$= 111 \times (111)^{221} \times [888^{111} + 9^{111}]$

Now, we know that 111 = 37 × 3. Also, the expression $111^{222} \times [888^{111} + 9^{111}]$ is definitely odd.

Therefore, we can say that the given expression is divisible by 37 and 3, but not 2.

Alternate method:

Here, we have 2 numbers out of which 333^{222} is odd, while 222^{333} is even.

Now, we know that the sum of an odd and an even number is always an odd number.

Therefore, the given expression is not divisible by 2.

Hence, option (b) is correct.

Q.31 What is the rightmost digit preceding the zeros in the value of 30^{30}?

(a) 1 (b) 3

(c) 7 (d) 9

[UPSC-2024]

Sol. (d)

$(30)^{30} = (10 \times 3)^{30} = 3^{30} \times 10^{30}$

Since, 10^{30} will result in a 31-digit number with 30 zeros at the end. So, the rightmost digit preceding the zeros will be the unit digit of 3^{30}.

Now, $3^{30} = 3^{(28+2)} = 3^{(4\times7+2)}$

We know that $3^1 = 3$; $3^2 = 9$; $3^3 = 27$, i.e. unit digit 7; $3^4 = 81$, i.e. unit digit 1; $3^5 = 3$; and so on. So, we can say that after every 4th power the unit digit will repeat, i.e. 3 has a cyclicity of 4.

So, the unit digit of $3^{(4\times7+2)}$ [which is of the form $3^{(4n+2)}$] must be 9.

Hence, the rightmost digit preceding the zeros will be 9.

Q.32 421 and 427, when divided by the same number, leave the same remainder 1. How many numbers can be used as the divisor in order to get the same remainder 1?

(a) 1 (b) 2

(c) 3 (d) 4

[UPSC-2024]

Sol. (c)

Here, the numbers are 421 and 427

When we subtract 421 and 427 by 1, we get 420 and 426.

Now by taking the prime factors we get:

420 = 2×2×3×5×7×1

426 = 2×3×71×1

The common factors among them are 2, 3, and 6 (multiple of 2 and 3).

Hence, we can say that 3 numbers can be used as the divisor.

Q.33 Consider the following statements in respect of the sum S = x + y + z, where x, y and z are distinct prime numbers each less than 10:

1. The unit digit of S can be 0.
2. The unit digit of S can be 9.
3. The unit digit of S can be 5.

Which of the statements given above are correct?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

[UPSC-2024]

Sol. (c)

x, y and z are distinct prime numbers less than 10. So, these numbers can be 2, 3, 5, or 7.

S = x + y + z

S = 2 + 3 + 5 = 10 ...(1)

S = 3 + 5 + 7 = 15 ...(3)

So, (2,3,5) and (3,5,7) satisfy conditions 1 and 3 respectively. However, statement 2 is not true for these given set of numbers.

Hence, option (c) is correct.

Q.34 Let X be a two-digit number and Y be another two-digit number formed by interchanging the digits of X. If (X+Y) is the greatest two-digit number, then what is the number of possible values of X?

(a) 2 (b) 4

(c) 6 (d) 8

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Sol. (d)

Let X = 10a + b and Y = 10b + a (interchanging the digits of X)

Greatest two-digit number = 99

According to the question,

X + Y = 99

Or 10a + b + 10b + a = 99

Or 11a + 11b = 99

Or 11(a + b) = 99

Or a + b = 9

Possible values of a and b:

a	*b*
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1

Value of X can be anything between 1 to 8.

Q.35 Let p, q, r and s be distinct positive integers. Let p, q be odd and r, s be even. Consider the following statements:

1. $(p - r)^2 (qs)$ is even.
2. $(q - s)q^2s$ is even.
3. $(q + r)^2 (p + s)$ is odd.

Which of the statements given above are correct?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

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Sol. (d)

p and q are odd numbers.

Similarly, r and s are even numbers.

Statement 1:

$(p - r)^2(qs)$ is even, as s is an even number.

This statement is correct.

Statement 2:

$(q - s)q^2s$ is even, as s is an even number.

This statement is correct.

Statement 3:

$(q + r)^2(p + s)$ is odd, as both (q + r) and (p + s) are odd.

This statement is correct.

Q.36 What is the number of fives used in numbering a 260-page book?

(a) 55 (c) 57

(b) 56 (d) 60

[UPSC-2024]

Sol. (b)

From 1 to 100, 5 comes 20 times.

Similarly, from 101 to 200, 5 comes 20 times.

Similarly, from 201 to 300, 5 comes 20 times.

So, we just need to check the numbers after 260. Such numbers having 5 are 265, 275, 285, 295 i.e. 4 numbers.

So, the number of 5's used from 1 to 260

= (20 + 20 + 20) - 4 = 56

Hence, option (b) is correct.

Q.37 If the sum of the two-digit numbers AB and CD is the three-digit number 1CE, where the letters A, B, C, D, E denote distinct digits, then what is the value of A?

(a) 9

(b) 8

(c) 7

(d) Cannot be determined due to insufficient data

[UPSC-2024]

Sol. (a)

Case 1: There's no carry over from B+D

A + C = 1C

Or A + C = 10 + C

Or A = 10 (not possible)

Case 2: There's carry over 1 from B+D

A + C + 1 = 1C

Or A + C + 1 = 10 + C

Or A = 9

Hence, option (a) is correct.

Q.38 Three numbers x, y, z are selected from the set of the first seven natural numbers such that $x > 2y > 3z$. How many such distinct triplets (x, y, z) are possible?

(a) One triplet (b) Two triplets

(c) Three triplets (d) Four triplets

[UPSC-2024]

Sol. (d)

4 triplets satisfying the given conditions are: (5,2,1), (6,2,1), (7,2,1) and (7,3,1)

Q.39 $32^5 + 2^{27}$ is divisible by

(a) 3 (b) 7

(c) 10 (d) 11

[UPSC-2024]

Sol. (c)

$32^5 + 2^{27} = (2^5)^5 + 2^{27}$ [$\because (a^m)^n = a^{mn}$]

$= 2^{25} + 2^{27}$

$= 2^{25}(1 + 2^2)$

$= 2^{25} \times 5$

$= 2^{24} \times 2^1 \times 5$ ($\because a^{m+n} = a^m \times a^n$)

$= 2^{24} \times 10$

Thus, $32^5 + 2^{27}$ is completely divisible by 10.

Q.40 Let p and q be positive integers satisfying $p < q$ and $p + q = k$. What is the smallest value of k that does not determine p and q uniquely?

(a) 3 (b) 4

(c) 5 (d) 6

[UPSC-2024]

Sol. (c)

it is given that p and q are positive integers, such that p < q.

p + q = k

Case 1: If k = 3

p + q = 3

1 + 2 = 3

Here, p and q have unique values.

Hence, this is discarded.

Case 2: If k = 4

p + q = 4

1 + 3 = 4

Here, p and q have unique values.

Hence, this is discarded.

Case 3: If k = 5

p + q = 5

1 + 4 = 5

2 + 3 = 5

Thus, here p can be 1 or 2 and q can be 4 or 3.

So, here p and q have multiple possible values.

Q.41 A can X contains 399 litres of petrol and a can Y contains 532 litres of diesel. They are to be bottled in bottles of equal size so that whole of petrol and diesel would be separately bottled. The bottle capacity in terms of litres is an integer. How many different bottle sizes are possible?

(a) 3 (b) 4

(c) 5 (d) 6

[UPSC-2024]

Sol. (b)

Quantity of petrol contained in X = 399 litres

Quantity of diesel contained in Y = 532 litres

By taking the factors of 399 and 532 we get:

399 = 1×3×7×19

532 = 1×2×2×7×19

So, the possible bottle sizes are 1 litre, 7 litres, 19 litres and 7 × 19 = 133 litres.

Therefore, the number of possible bottle sizes is 4.

Q.42 What is the unit digit in the multiplication of 1 × 3 × 5 × 7 × 9 ×... × 999?

(a) 1 (b) 3

(c) 5 (d) 9

[UPSC-2025]

Sol. (c)

Let's consider the effect of multiplying a number ending in 5 by other numbers:

If a number ending in 5 is multiplied by any odd number, the unit digit of the result will be 5. For example:

5 × 1 = 5 (unit digit 5)

5 × 3 = 15 (unit digit 5)

5 × 7 = 35 (unit digit 5)

5 × 9 = 45 (unit digit 5)

$5 \times 11 = 55$ (unit digit 5)

$5 \times 13 = 65$ (unit digit 5)

If a number ending in 5 is multiplied by any even number, the unit digit of the result will be 0.

For example.

$5 \times 2 = 10$ (unit digit 0)

$5 \times 4 = 20$ (unit digit 0)

$5 \times 6 = 30$ (unit digit 0)

$5 \times 8 = 40$ (unit digit 0)

In given product

$1 \times 3 \times 5 \times 7 \times 9 \times \dots\dots\dots \times 999$

$1 \times 3 \times 5 \times 7 \times 9 \times \dots \times 999$, we are multiplying 5 by a series of other odd numbers (1, 3, 7, 9, 11, 13, ..., 999). Since 5 is being multiplied exclusively by odd numbers (and itself, which is odd), the unit digit of the final product must be 5.

Thus, the unit digit in the multiplication of $1 \times 3 \times 5 \times 7 \times 9 \times \dots \times 999$ is 5.

Hence option (c) is correct.

Q.43 If $N^2 = 12345678987654321$, then how many digits does the number N have?

(a) 8 (b) 9

(c) 10 (d) 11

[UPSC-2025]

Sol. (b)

The pattern of squares of numbers consisting only of ones:

$1^2 = 1$ (1 digit in N, 1 digit in N^2, highest digit in N^2 is 1)

$11^2 = 121$ (2 digits in N, 3 digits in N^2, highest digit in N^2 is 2)

$111^2 = 12321$ (3 digits in N, 5 digits in N^2, highest digit in N^2 is 3)

$1111^2 = 1234321$ (4 digits in N, 7 digits in N^2, highest digit in N^2 is 4)

$11111^2 = 123454321$ (5 digits in N, 9 digits in N^2, highest digit in N^2 is 5) …… and so on.

In this case,

$N^2 = 12345678987654321$. The highest digit in the sequence is 9, and the number of digits in it is 17. This means that N must be a number consisting of nine '1's. So, N = 111,111,111.

Thus, the number N has 9 digits.

Hence, option (b) is correct.

Q.44 A natural number N is such that it can be expressed as N = p + q + r, where p, q and r are distinct factors of N. How many numbers below 50 have this property?

(a) 6 (b) 7

(c) 8 (d) 9

[UPSC-2025]

Sol. (c)

We need to find natural numbers below 50 that satisfy:

N = p + q + r, where p, q, and r are distinct factors of N (excluding N itself).

The smallest such number is (1 + 2 + 3 = 6).

Now checking multiples of 6:

Number N	*Factors of N*	*Distinct Factors p, q, r such that $N = p + q + r$*
6	1, 2, 3, 6	$1+2+3=6$
12	1, 2, 3, 4, 6, 12	$2+4+6=12$
18	1, 2, 3, 6, 9, 18	$3+6+9=18$
24	1, 2, 3, 4, 6, 8, 12, 24	$4+8+12=24$
30	1, 2, 3, 5, 6, 10, 15, 30	$5+10+15=30$
36	1, 2, 3, 4, 6, 9, 12, 18, 36	$6+12+18=36$
42	1, 2, 3, 6, 7, 14, 21, 42	$7+14+21=42$
48	1, 2, 3, 4, 6, 8, 12, 16, 24, 48	$8+16+24=48$

Thus, the numbers below 50 that satisfy the given condition are: 6, 12, 18, 24, 30, 36, 42, 48, i.e. eight in total.

Hence, option (c) is correct.

Q.45 Three prime numbers p, q and r, each less than 20, are such that p – q = q – r. How many distinct possible values can we get for (p+q+r)?

(a) 4 (b) 5

(c) 6 (d) More than 6

[UPSC-2025]

Sol. (a)

Three prime numbers p, q and r, each less than 20, are such that p – q = q – r

Here, p – q = q – r

Or, 2q = p + r

So, p, q and r are in arithmetic progression.

There are 8 prime numbers under 20: 2, 3, 5, 7, 11, 13, 17 and 19.

Down below are all such combinations wherein three such primes form an A.P.:

1. If q = 5, then p and r can be either 3 or 7. Sum of p + q + r = 3 + 5 + 7 = 15.
2. If q = 7, then p and r can be either 3 or 11. Sum of p + q + r = 3 + 7 + 11 = 21.
3. If q = 11, then p and r can be either 5 or 17. Sum of p + q + r = 5 + 11 + 17 = 33.
4. If q = 11, then p and r can also be either 3 or 19. Sum of p + q + r = 3 + 11 + 19 = 33.
5. If q = 13, then p and r can be either 7 or 19. Sum of p + q + r = 7 + 13 + 19 = 39.

Thus, there are 4 distinct possible values of (p + q + r).

Hence, option (a) is correct.

Q.46 How many possible values of (p + q + r) are there satisfying $\frac{1}{p}+\frac{1}{p}+\frac{1}{r}=1$, where p, q and r are natural numbers (not necessarily distinct)?

(a) None (b) One

(c) Three (d) More than three

[UPSC-2025]

Sol. **(c)**

$\frac{1}{p}+\frac{1}{q}+\frac{1}{r}$ = 1, where p, q, and r are natural numbers (not necessarily distinct).

Possible values of p, q and r are:

1. $\frac{1}{4}+\frac{1}{4}+\frac{1}{2}=1$
2. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$
3. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$

(p, q, r)	$p+q+r$
(4, 4, 2)	$4+4+2=10$
(2, 3, 6)	$2+3+6=11$
(3, 3, 3)	$3+3+3=9$

Thus, there are 3 distinct possible values of (p + q + r) satisfying $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=1$.

Hence, option (c) is correct.

Q.47 Let both p and k be prime numbers such that (p^2 + k) is also a prime number less than 30.

What is the number of possible values of k?

(a) 4 (b) 5

(c) 6 (d) 7

[UPSC-2025]

Sol. **(b)**

Given: p and k are prime numbers. Also, (p^2 + k) is a prime number less than 30.

Prime numbers less than 30 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

We basically need to find all possible pairs of prime numbers (p) and (k), such that (p2 + k) is a prime number less than 30.

Since $p^2 + k < 30$, the possible values that p may attain are 2, 3, or 5. If $p \geq 6$, p^2 + k would be more than 36.

<u>If p = 2:</u>

$p^2 + k = 4 + k$

Possible values for k are: 2, 3, 5, 7, 11, 13, 17, 19, 23

Let's calculate (p^2 + k) for each value of k:

(4 + 2 = 6) (not prime)

(4 + 3 = 7) (prime)

(4 + 5 = 9) (not prime)

(4 + 7 = 11) (prime)

(4 + 11 = 15) (not prime)

(4 + 13 = 17) (prime)

(4 + 17 = 21) (not prime)

(4 + 19 = 23) (prime)

(4 + 23 = 27) (not prime)

Hence, possible values for k when p = 2 are: 3, 7, 13, 19.

<u>If p = 3:</u>

Possible values for k are: 2, 3, 5, 7, 11, 13, 17, 19

Let's calculate (p^2 + k) for each value of k:

(9 + 2 = 11) (prime number)

Since, all the remaining values of k are odd, the sum (p^2 + k) would be an even number (So, cannot be prime).

Hence, the only possible value for k when p = 3 is: 2.

<u>If p = 5:</u>

Possible values for k are: 2, 3

Let's calculate (p^2 + k) for each value of k:

(25 + 2 = 27) (not prime)

(25 + 3 = 28) (not prime)

So, no possible value for k exists when p = 5.

Therefore, the number of possible values of k = 4 + 1 = 5

Hence, option (b) is correct.