

NEXT IAS

(To be filled by candidate)

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Date of Examination :

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31/7

Exam Centre : Old Rajinder Nagar ☐Online ☒**CSE (MAINS) TEST SERIES-2024**Test ~~4~~ 6**MATHEMATICS
OPTIONAL**

Dated : 14-07-2024

Time Allowed : Three Hours

Maximum Marks: 250

QUESTION PAPER SPECIFIC INSTRUCTIONS*(Please read each of the following instructions carefully before attempting questions)**There are EIGHT questions divided in Two sections and printed in ENGLISH.**Candidate has to attempt FIVE questions in all.**Question No. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each section.**The number of marks carried by a question/part is indicated against it.**Answers must be written in the medium authorized in the Admission Certificate which must be stated clearly on the cover of this Question cum Answer (QCA) booklet in the space provided.**No marks will be given for answers written in a medium other than the authorized one.**Word limit in questions, wherever specified, should be adhered to.**Attempts of question shall be counted in sequential order, unless struck off. Attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question cum Answer (QCA) booklet must be clearly struck off.***DELHI CENTRE:**

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(For filling by Examiners only)

Q. No.	Page No.	Max. Marks	Marks	Total	Signature
1. (a)	4	10			
1. (b)	6	10			
1. (c)	8	10			
1. (d)	10	10			
1. (e)	12	10			
2. (a)	14	15			
2. (b)	17	12			
2. (c)	19	13			
2. (d)	21	10			
3. (a)	23	15			
3. (b)	26	15			
3. (c)	29	20			
4. (a)	33	15			
4. (b)	36	15			
4. (c)	39	20			
5. (a)	43	10			
5. (b)	45	10			
5. (c)	47	10			
5. (d)	49	10			
5. (e)	51	10			
6. (a)	53	18			
6. (b)	57	15			
6. (c)	60	17			
7. (a)	64	20			
7. (b)	68	15			
7. (c)	71	15			
8. (a)	74	15			
8. (b)	77	15			
8. (c)	80	20			

Remarks

Observations:

1. The first observation is that the system is stable and the response is linear. The input signal is a step function and the output signal is a step function. The system is stable and the response is linear. The input signal is a step function and the output signal is a step function.

2. The second observation is that the system is stable and the response is linear. The input signal is a step function and the output signal is a step function. The system is stable and the response is linear. The input signal is a step function and the output signal is a step function.

3. The third observation is that the system is stable and the response is linear. The input signal is a step function and the output signal is a step function. The system is stable and the response is linear. The input signal is a step function and the output signal is a step function.

4. The fourth observation is that the system is stable and the response is linear. The input signal is a step function and the output signal is a step function. The system is stable and the response is linear. The input signal is a step function and the output signal is a step function.

5. The fifth observation is that the system is stable and the response is linear. The input signal is a step function and the output signal is a step function. The system is stable and the response is linear. The input signal is a step function and the output signal is a step function.

SECTION-A

- Q.1 (a) Let R be a commutative ring with unity such that $a^2 = a, \forall a \in R$. If I be any prime ideal of R , find all the elements of R/I . (10)

$\frac{R}{I}$ is integral domain

$$(I + r_1) ; (I + r_2) \in \frac{R}{I}$$

$$(I + r_1)^2 = I + r_1^2 = (I + r_1)$$

$$(I + r_1) [(I + r_1) - (I + e)] = 0$$

$$I + r_1 = 0$$

$$I + (r_1 - e) = 0$$

$$r_1 - e \in I$$

$$\nexists I + r_1 = 0$$

$$r_1 = -\alpha (\alpha \in I)$$

$$I + (r_1 - e) = 0$$

$$r_1 - e \in I$$

$$r_1 \in e + I$$

$$I + r_1 = 0 ; I + r_1 = e$$

$\Rightarrow 0, e$ are only two elements.

Q.1 (b)

Determine for each of the sets $\left\{1, 1 + \frac{(-1)^n}{10^n} : n \in \mathbb{N}\right\}$, $\left\{1 - \frac{2}{n} : n \in \mathbb{N}\right\}$, $\left\{\frac{m}{n} : m, n \in \mathbb{N}\right\}$

I. The suprema and infima

II. The limit points

III. The lower and upper limits. Specify which of the above sets are closed or open? (10)

I. ^{Suprema}

$$\left\{1, 1 + \frac{(-1)^n}{10^n}\right\} = 1 + \frac{1}{100}$$

$$\left\{1 - \frac{2}{n}\right\} = 1$$

$$\left\{\frac{m}{n}\right\} = m \text{ (very large)}$$

^{Infima}

$$\left\{1, 1 + \frac{(-1)^n}{10^n}\right\} = 1 - \frac{1}{10}$$

$$\left\{1 - \frac{2}{n}\right\} = -1$$

$$\left\{\frac{m}{n}\right\} = 0$$

II limit points

$$\left\{1, 1 + \frac{(-1)^n}{10^n}\right\} = 1$$

$$\left\{1 - \frac{2}{n}\right\} = 1$$

$$\left\{\frac{m}{n}\right\} = \mathbb{R}^+$$

III

$$\left\{ 1, 1 + \frac{(-1)^n}{10^n} \right\}$$

$$\text{Lower limit} = 1$$

$$\text{Upper limit} = 1$$

$$\left\{ 1 - \frac{2}{n} \right\}$$

$$\text{both} = 1$$

$$\left\{ \frac{m}{n} \right\}$$

$$\text{Upper limit} = \text{---}$$

$$\text{Lower limit} = 0$$

IV $S_1,$ S_3

are closed set

as

limit point $(1) \in \text{set.}$

Q.1 (c)

Show that $1 - \frac{1}{4 \cdot 3} + \frac{1}{4^2 \cdot 5} - \frac{1}{4^3 \cdot 7} + \dots$ is convergent.

(10)

$$u_n = \frac{(-1)^n}{4^n \cdot (2n+1)}$$

Neglecting 1; $u_1 = \frac{-1}{4 \cdot 3}$

$$\left| \frac{u_n}{u_{n+1}} \right| = \left| \frac{(-1)^n \cdot 4(2n+3)}{(2n+1)} \right| > 1$$

$$\Rightarrow |u_n| > |u_{n+1}|$$

 ~~$\Rightarrow$ convergent by ratio test~~

Also $\lim_{n \rightarrow \infty} u_n = 0$

By Leibniz test

 u_n converges.

If $v(r, \theta) = \left(r - \frac{1}{r}\right) \sin \theta$, $r \neq 0$, then find analytic function $f(z) = u(r, \theta) + iv(r, \theta)$. (10)

$$\frac{\partial v}{\partial r} = \left(1 + \frac{1}{r^2}\right) \sin \theta$$

$$\frac{\partial v}{\partial \theta} = \left(r - \frac{1}{r}\right) \cos \theta.$$

$$f(z)$$

$$\frac{df}{dz} = -i \frac{\partial f}{\partial \bar{y}} = -i \left(\frac{\partial f}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial f}{\partial \theta} \frac{\partial \theta}{\partial y} \right)$$

$$= -i \frac{\partial f}{\partial r} \left(\frac{\partial r}{\partial y} + i r \frac{\partial \theta}{\partial y} \right)$$

$$= -i \frac{\partial f}{\partial r} \left(\frac{y}{r} + i r \left(\frac{-y}{x^2 + y^2} \right) \right)$$

~~f~~

$$f = u + iv$$

$$\frac{\partial f}{\partial r} = u_r + i v_r = \frac{1}{r} (v_\theta - i u_\theta) = \frac{-i f_\theta}{r}$$

$$\frac{\partial f}{\partial \theta} = u_\theta + i v_\theta = -v_r + i v_\theta$$

$$\frac{\partial \theta}{\partial y} = \frac{\partial \tan^{-1} y/x}{\partial x} = \frac{\left(\frac{-y}{x^2} \right)}{1 + \frac{y^2}{x^2}} = \frac{-y}{x^2 + y^2}$$

$$\frac{df}{dz} = \sin \theta (1-i) \frac{\partial f}{\partial r}$$

$$\frac{df}{dz} = \sin \theta (1-i) \left[\left(1 - \frac{1}{r^2}\right) \cos \theta + i \left(1 + \frac{1}{r^2}\right) \sin \theta \right]$$

$$\frac{\partial f}{\partial r} = \left(1 - \frac{1}{r^2}\right) \cos \theta + i \left(1 + \frac{1}{r^2}\right) \sin \theta$$

$$f = \left(r + \frac{1}{r}\right) \cos \theta + i \left(r - \frac{1}{r}\right) \sin \theta + C$$

$$f = (\cos \theta + i \sin \theta) + \frac{1}{r} (\cos \theta - i \sin \theta)$$

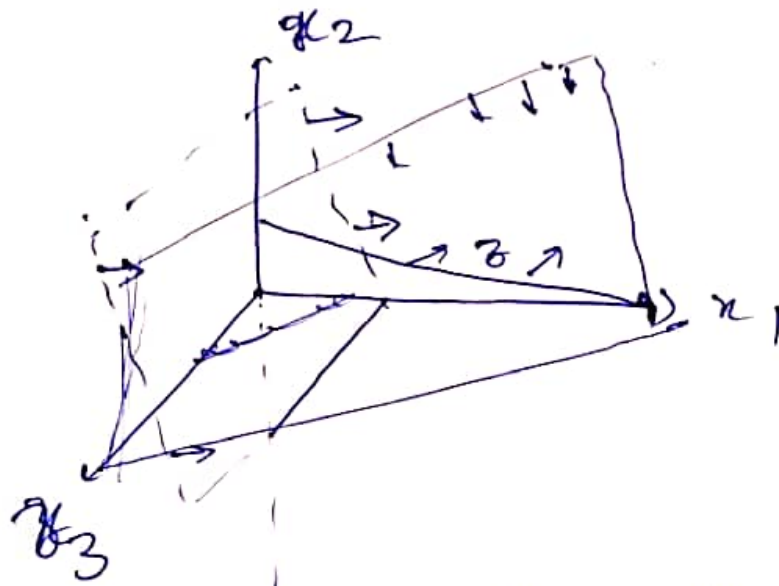
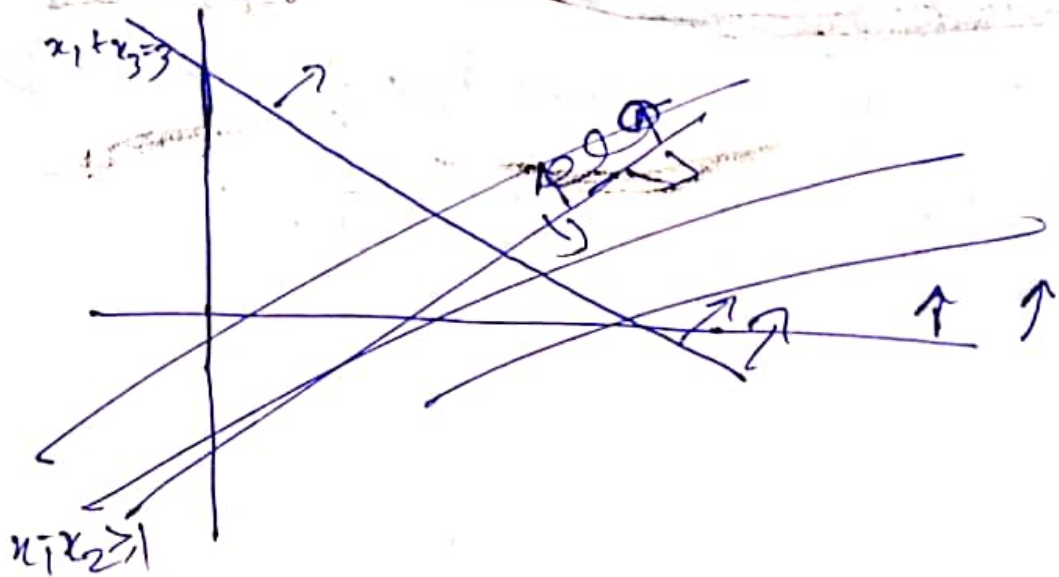
$$f = z + \frac{1}{z} + C$$

Q. (a) Using graphical method, solve the linear programming problem.

Maximise $Z = 3x_1 + 5x_2$

Subject to $x_1 - x_2 \geq 1$
 $x_1 + x_2 \geq 3$
 $x_1, x_2 \geq 0$

(10)



unbounded

Let $R = \mathbb{Z}[i] = \{a+ib \mid a, b \in \mathbb{Z}\}$. Let $M = \langle 2+i \rangle$ then show that M is maximal ideal of R .

By contradiction

(15)

$$M \subsetneq U \subsetneq R$$

$$\langle 2+i \rangle \subsetneq \langle a+bi \rangle \subsetneq R$$

$$(2+i) = (a+bi)(c+di)$$

$$(2-i) = (a-bi)(c-di)$$

$$5 = (a^2+b^2)(c^2+d^2)$$

$$a^2+b^2=1$$

$$c^2+d^2=5$$

$$\text{If } a^2+b^2=1$$

$$(a,b) = (\pm 1, 0) \text{ or } (0, \pm 1)$$

$$\text{then } \langle a+bi \rangle = R$$

$$U = R$$

$$a^2+b^2=5$$

$$c^2+d^2=1$$

$$\text{If } c^2+d^2=1$$

$$(c,d) = (\pm 1, 0) \text{ or } (0, \pm 1)$$

$$\Rightarrow (2+i) = \pm(a+bi)$$

$$\text{or } (2+i) = \pm i(a+bi)$$

$$\Rightarrow (a+bi) = \pm(2+i)$$

$$\text{or } \cancel{\pm(2-i)} \\ \pm i(2+i)$$

$$\Rightarrow (a+bi) \in \langle 2+i \rangle$$

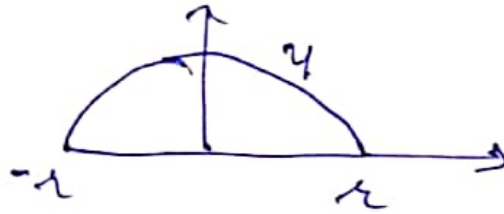
$$\Rightarrow U \subsetneq M$$

$$\Rightarrow U \subsetneq M$$

$\Rightarrow M$ is maximal.

Prove by contour integration $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx = \pi \ln 2.$

(15)



C is semi circle

$$\int_C \frac{\log(i+z)}{1+z^2} dz$$

~~Residue~~ inside C is $z = i$

$$\Rightarrow \int_C f = 2\pi i \frac{\log(2i)}{2i}$$

$$\int_C f(z) dz = \lim_{R \rightarrow \infty} \int_{-R}^R f(z) dz + \int_{\gamma} f(z) dz$$

$$\pi \log 2i = \lim_{R \rightarrow \infty} \int_{-R}^R f(x) dx + 0$$

($\because \lim_{z \rightarrow \infty} z f(z) = 0$ Jordan lemma)

$$\pi \log 2i = \int_{-\infty}^{\infty} \frac{\log(i+x)}{1+x^2} dx$$

~~$$\int_{-\infty}^{\infty} \frac{\log x^2 + 1}{2x^2 + 1} dx = \int_{-\infty}^{\infty} \frac{\log x^2}{2x^2 + 1} dx + \int_{-\infty}^{\infty} \frac{1}{2x^2 + 1} dx$$~~

Solve the following linear Programming problem using Big M method: (20)

Maximise $Z = 4x_1 + 5x_2 + 2x_3$

$2x_1 + x_2 + x_3 \geq 10,$

$x_1 + 3x_2 + x_3 \leq 12$

Subject to: $x_1 + x_2 + x_3 = 6$

$x_1, x_2, x_3 \geq 0$

Max $Z = 4x_1 + 5x_2 + 2x_3 + 0e_1 + 0s_1 - Ma_1 - Ma_2$

s.c $2x_1 + x_2 + x_3 - e_1 + a_1 = 10$

$x_1 + 3x_2 + x_3 + s_1 = 12$

$x_1 + x_2 + x_3 + a_2 = 6$

$x_1, x_2, x_3, a_1, a_2, s_1, e_1 \geq 0$
 $a_1 = 10; s_1 = 12; a_2 = 6$

JBFS

C_B	X_B	x_1	x_2	x_3	e_1	s_1	a_1	a_2	b	θ
-M	a_1	2	1	1	-1	0	1	0	10	5
0	s_1	1	3	1	0	1	0	0	12	12
-M	a_2	1	1	1	0	0	0	1	6	6
Z_j		-3M	-2M	-2M	M	0	-M	-M	-16M	
C_j		4+3M	5+2M	2+2M	-M	0	0	0		
4	x_1	1	1/2	1/2	-1/2	0	1/2	0	5	10
0	s_1	0	5/2	1/2	1/2	1	-1/2	0	7	14/5
-M	a_2	0	1/2	1/2	1/2	0	-1/2	1	1	2
Z_j		4	$2 - \frac{M}{2}$	$2 - \frac{M}{2}$	$-2 - \frac{M}{2}$	0	$2 + \frac{M}{2}$	-M	-2M	
C_j		0	$3 + \frac{M}{2}$	$\frac{M}{2}$	$2 + \frac{M}{2}$	0	$-\frac{3M}{2} - 2$	0		
4	x_1	1	0	0	-1	0	1	-1	4	6
0	s_1	0	0	-2	-2	1	2	-5	2	
5	x_2	0	1	1	1	0	-1	1	2	
Z_j		4	5	-5	-1	0	-1	-1	-2	
C_j		0	0	-3	-1	0	-M+1	-M+1		

$$x_1 = 4, x_2 = 2, x_3 = 0$$

$$\text{All } c_j \leq 0$$

optimally reached

$$z_{\max} = 26.$$

SECTION-B

Construct a partial differential equation of all surfaces of revolution having the z -axis as the axis of rotation. (10)

Candidates must not write on this margin

$$z = \phi(x^2 + y^2)$$

$$\frac{\partial z}{\partial x} = 2x \phi'(x^2 + y^2)$$

$$\frac{\partial z}{\partial y} = 2y \phi'(x^2 + y^2)$$

$$\boxed{x \frac{\partial z}{\partial y} = y \frac{\partial z}{\partial x}}$$

Find the order of convergence of Regula-Falsi Method.

(10)

Candidates
must not write
on this margin

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$



$$A' + AB = 0$$

$$(A' + A)(A' + B) = 0$$

$$A' + B = 0 \Rightarrow A' = B = 0$$

$$\Rightarrow A = 1; B = 0$$

$$AB = AC \Rightarrow C = 0$$

$$AB + AC' + CD = C'D$$

$$0 + 1 + 0 = D$$

$$\Rightarrow D = 1$$

$$(A, B, C, D) = (1, 0, 0, 1)$$

Find the values of the two valued variables A, B, C and D by solving the set of simultaneous equations $A' + AB = 0$, $AB = AC$, $AB + AC' + CD = C'D$. (10)

Candidates must not write on this margin

$$A' + AB = 0$$

$$(A' + A)(A' + B) = 0$$

$$A' + B = 0 \Rightarrow A' = B = 0$$

$$\Rightarrow A = 1; B = 0$$

$$AB = AC \Rightarrow C = 0$$

$$AB + AC' + CD = C'D$$

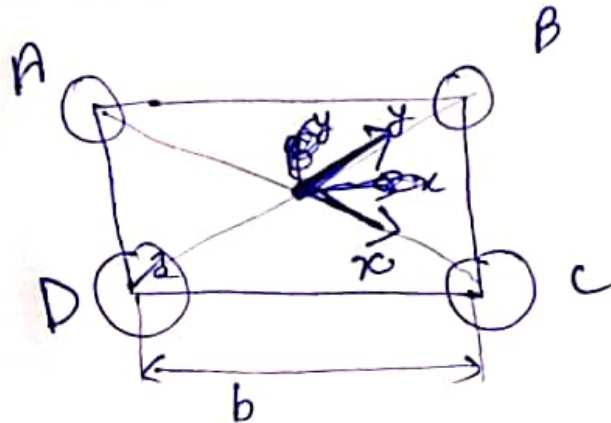
$$0 + 1 + 0 = D$$

$$\Rightarrow D = 1$$

$$(A, B, C, D) = (1, 0, 0, 1)$$

Four solid sphere A, B, C and D, each of mass m and radius a , are placed with their centres on the four corners of a square of side b . Calculate the moment of inertia of the system about a diagonal of the square. (10)

Candidates must not write on this margin



$$I_3 = 4m\left(\frac{b}{\sqrt{2}}\right)^2 + 4m\left(\frac{2}{5}a^2\right)$$

$$I_x + I_y = I_3$$

(Diagonal of square at 90°)

also $I_x = I_y$ (symmetry)

$$\Rightarrow 2I_x = 4m\left(2b^2 + \frac{8}{5}a^2\right)$$

$$I_x = mb^2 + \frac{4ma^2}{5}$$

$$I_{yA} = I_{yC} = \frac{2}{5}ma^2 + m\left(\frac{b}{\sqrt{2}}\right)^2$$

$$I_{yB} = I_{yD} = \frac{2}{5}ma^2$$

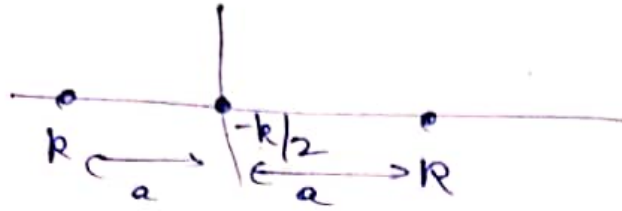
$$I_y = I_{yA} + I_{yB} + I_{yC} + I_{yD}$$

$$I_y = \frac{8}{5}ma^2 + mb^2 = I_x$$

x, y are along diagonal

Two point vortices each of strength k are situated at $(\pm a, 0)$ and a point vortex of strength $-k/2$ is situated at the origin. Show that the fluid motion is stationary and find the equations of streamlines.

(10)



w at $(0, 0)$ due to other 2 vortices

$$w = \frac{ik}{2\pi} \log(z-a) + \frac{ik}{2\pi} \log(z+a)$$

$$\frac{dw}{dz} = \frac{ik}{2\pi} \left(\frac{1}{z-a} + \frac{1}{z+a} \right)$$

$$\left. \frac{dw}{dz} \right|_{z=0} = 0 \quad \text{hence vortex is stationary}$$

$\therefore$ fluid motion is stationary

$$w = \frac{ik}{2\pi} \left[\log(z-a) + \log(z+a) - \frac{1}{2} \log z \right]$$

$$w = \phi + i\psi$$

$$\psi = \frac{k}{2\pi} \frac{1}{2} \log \frac{(\sqrt{(x-a)^2 + y^2})(\sqrt{(x+a)^2 + y^2})}{\sqrt{x^2 + y^2}}$$

$$\psi = \frac{Ck}{4\pi}$$

$$\Rightarrow \frac{((x-a)^2 + y^2)((x+a)^2 + y^2)}{\sqrt{x^2 + y^2}} = C$$

Find the integral surface of the linear partial differential equation

$$x(y^2+z)\frac{\partial z}{\partial x} - y(x^2+z)\frac{\partial z}{\partial y} = (x^2-y^2)z \text{ through the straight line } x+y=0, z=1. \quad (18)$$

$$P + qQ = R$$

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{(x^2-y^2)z}$$

$$= \frac{x dx + y dy + \cancel{z dz} - dz}{x^2 z - y^2 z - x^2 z + y^2 z}$$

$$\Rightarrow d\left(\frac{x^2}{2} + \frac{y^2}{2} - z\right) = 0$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} - z = c_1$$

$$z = \frac{x^2+y^2}{2} - c_1$$

$$\frac{dz}{x(y^2 + \frac{x^2+y^2}{2} - c_1)} = \frac{dy}{-y(x^2 + \frac{x^2+y^2}{2} - c_1)}$$

$$\frac{dz}{x(3y^2 + x^2 - 2c_1)} = \frac{dy}{-y(3x^2 + y^2 - 2c_1)}$$

$$\frac{dy}{dx} = \frac{-y(3x^2 + y^2 - 2c_1)}{x(3y^2 + x^2 - 2c_1)}$$

$$dx + dy + dz$$

$$xy(y-z) + z(x-y) + z(x-y)(x+y)$$

$$= dx + dy + dz$$

$$(x-y)(-xy + z) + z(x+y) = dx - dy - dz$$

$$= \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$$

0

$$\Rightarrow \log xyz = \log C_2$$

$$\boxed{\begin{aligned} xyz &= C_2 \\ x^2 + y^2 - 2z &= C_3 \end{aligned}}$$

$$x+y=0; z=1$$

$$x = -y = k$$

$$-k^2 = C_2; \quad 2k^2 - 2 = C_3$$

$$-2C_2 - 2 = C_3$$

$$(\cancel{C_2} + 1) \quad C_3 + 2C_2 = -2$$

$$\boxed{(x^2 + y^2 - 2z) + 2(xyz) = -2}$$

Given $f(1) = 4, f(2) = 5, f(7) = 5$ and $f(8) = 4$. Find the value of $f(6)$ and also the value of x for which $f(x)$ is maximum or minimum.

(15)

$$f(x) = \frac{(x-1)(x-7)(x-8)}{(-1)(-6)(-7)} \cdot 4 + \frac{(x-1)(x-7)(x-8)}{(1)(-5)(-6)} \cdot 5$$

$$+ \frac{(x-1)(x-2)(x-8)}{6(5)(-1)} \cdot 5 + \frac{(x-1)(x-2)(x-7)}{7(6)(1)} \cdot 4$$

$$f(x) = x^3 \left[\frac{4}{-42} + \frac{5}{30} + \frac{5}{-30} + \frac{4}{42} \right]$$

$$+ x^2 \left[\frac{4(-17)}{-42} + \frac{5(-16)}{306} - \frac{1(-11)}{6} + \frac{4(-10)}{42} \right]$$

$$+ x \left[\frac{4(86)}{-42} + \frac{5(71)}{30} - \frac{1(26)}{6} + \frac{4(23)}{42} \right]$$

$$+ \left(\frac{4 \times 7 \times 2 \times 8}{+42 \times 3} - \frac{1 \times 7 \times 8 \times 5}{5 \times 6 \times 3} + \frac{5 \times 16}{30 \times 3} - \frac{7 \times 2 \times 4}{7 \times 6 \times 3} \right)$$

$$= x^2 \left(\frac{-28}{42} - \frac{5}{6} \right) + x \left(\frac{4 \times 63}{2 \times 42} + \frac{45}{62} \right)$$

$$+ \left(\frac{32 - 28 + 8 - 4}{3} \right)$$

$$\frac{-18}{7} \cdot \frac{5}{6} = \frac{28}{42} - \frac{5}{6}$$

$$f(x) = x^2 \left(-\frac{7}{42} \right) + x \left(\frac{3}{2} \right) + \frac{8}{3}$$

$$f(6) = -6 + 9 + \frac{8}{3} = \frac{17}{3}$$

$$f'(x) = 0 = -\frac{x}{3} + \frac{3}{2} \Rightarrow x = \frac{9}{2}$$

$x = \frac{9}{2}$ is maxima ($f''(x) < 0$)

For a dynamical system $T = \frac{1}{2}\{(1+2k)\dot{\theta}^2 + 2\dot{\theta}\dot{\phi} + \dot{\phi}^2\}$, $V = \frac{n^2}{2}\{(1+k)\theta^2 + \phi^2\}$, where θ, ϕ are coordinates and n, k are positive constants, write down the Lagrange's equations of motion and deduce that $(\theta - \phi) + n^2 \frac{1+k}{k}(\theta - \phi) = 0$. Further show that if $\theta = \phi, \dot{\theta} = \dot{\phi}$ at $t = 0$ then $\theta = \phi$ for all t .

(17)

$$L = T - V$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\phi}} \right) - \frac{\partial L}{\partial \phi} = 0$$

$$L = \dot{\theta}^2 \left(\frac{1+2k}{2} \right) - \left(\frac{n^2}{2}(1+k) \right) \theta^2 + \dot{\theta} \dot{\phi} + \frac{\dot{\phi}^2}{2} - \frac{\phi^2 n^2}{2}$$

$$\frac{\partial L}{\partial \theta} = -n^2(1+k)\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = \dot{\theta}(1+2k) + \dot{\phi}$$

$$\frac{\partial L}{\partial \phi} = -n^2\phi$$

$$\frac{\partial L}{\partial \dot{\phi}} = \dot{\theta} + \dot{\phi}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{d}{dt} \left(\dot{\theta}(1+2k) + \dot{\phi} \right) + n^2(1+k)\theta = 0$$

$$\ddot{\theta}(1+2k) + \ddot{\phi} + n^2(1+k)\theta = 0 \quad (i)$$

$\phi \text{ eq}^n$

$$\frac{d}{dt}(\dot{\theta} + \dot{\phi}) + n^2\phi = 0$$

$$\ddot{\theta} + \ddot{\phi} + n^2\phi = 0 \quad \text{---(ii)}$$

$$\ddot{\theta} = -(\ddot{\phi} + n^2\phi)$$

$$-(1+2k)(\ddot{\phi} + n^2\phi) + \ddot{\phi} + n^2(1+k)\theta = 0$$

$$\ddot{\phi} \cancel{(1+k)} = -\frac{n^2(1+k)}{+2k}\theta + \frac{(1+2k)n^2\phi}{+2k}$$

$$\ddot{\theta} = -(\ddot{\phi} + n^2\phi) = + \left(\frac{-2k}{-n^2(1+k)\theta + (1+2k)n^2\phi} \right)$$

$$\ddot{\theta} - \ddot{\phi} = \frac{-n^2(1+k)}{2k}(\cancel{\theta}) - \frac{\phi n^2(2+k)}{2k}$$

$$= \frac{n^2(1+k)}{k} \left[-\theta + \phi \right]$$

$$\ddot{\theta} - \ddot{\phi} + \frac{n^2(1+k)}{k}(\theta - \phi) = 0.$$

$$\cancel{\ddot{\theta} - \ddot{\phi}}$$

$$\cancel{\theta - \phi} \Rightarrow \theta - \phi = u$$

$$\ddot{u} = -\frac{n^2(1+k)}{k} u$$

$\Rightarrow$ SHM

$$u = A \sin \left(\sqrt{\frac{n^2(1+k)}{k}} t + \alpha \right)$$

$$\text{at } t=0 \quad u=0 \Rightarrow A \sin \alpha = 0$$

$$t=0 \quad \dot{u}=0 \Rightarrow A \cos \alpha = 0$$

$$\Rightarrow A = 0 \quad \theta = \phi = u = 0$$

Find a surface satisfying $x - 2y + z = 6$ and touching the hyperbolic paraboloid $z = xy$ along its section by the plane $y = x$. (20)

$$x - 2y + z = 6$$

$$(D^2 - 2DD' + D'^2)z = 6$$

$$(D - D')^2 z = 6$$

$$AE \rightarrow (m-1)^2 = 0 \Rightarrow m = 1, 1$$

$$z_c = \phi_1(y+x) + x \phi_2(y+x)$$

$$z_p = \frac{1}{(D - D')^2} 6$$

$$= \frac{1}{D^2 \left(1 - \frac{D'}{D}\right)^2} 6$$

$$= \frac{1}{D^2} \left(1 - \frac{D'}{D}\right)^{-2} 6$$

$$= \frac{1}{D^2} \left(1 + 2\frac{D'}{D} + \dots\right) 6$$

$$= \frac{1}{D^2} 6 = \frac{1}{D} 6x = 3x^2$$

$$z = z_p + z_c$$

$$= \phi_1(y+x)(1) + x \phi_2(y+x) + 3x^2$$

along $y = x = k$
 $z = k^2$

$$k^2 = \phi_1(2k) + k\phi_2(2k) + 3k^2$$

Also it touches

$$\Rightarrow p = \frac{\partial xy}{\partial x} \Big|_{x=k=y}$$

$$q = \frac{\partial xy}{\partial y} \Big|_{y=k=x}$$

$$p = k = q$$

$$p = \phi'_1(y+x) + \phi_2(y+x) + x\phi'_2(y+x)$$

$$q = \phi'_1(y+x) + x\phi'_2(y+x)$$

$$k = \phi'_1(2k) + \phi_2(2k) + k\phi'_2(2k) + 6k$$

$$k = \phi'_1(2k) + k\phi'_2(2k)$$

$$\phi_2(2k) + 6k = 0$$

$$\phi_2(2k) = -6k = -3(2k)$$

$$\phi_2(x) = -3x$$

$$\Rightarrow k^2 = \phi_1(2k) + k(-6k) + 3k^2$$

$$\phi_1(2k) = (2k)^2$$

$$\phi_1(x) = x^2$$

$$z = (y+x)^2 + x(-3)(x+y) + 3x^2$$

$$= x^2 + y^2 - 3xy$$

Q.7 (b)

In a 4 bit representation, what is the value of 1111 in signed integer form, unsigned integer form, signed 1's complement form and signed 2's complement form. (15)

in signed

$$(1111)_2 = - (111)_2 \\ = (-7)_{10}$$

in unsigned

$$(1111)_2 = (8+4+2+1)_{10} \\ = (15)_{10}$$

In signed 1st complement

$$\overline{1111} = 0000 = (0)_{10}$$

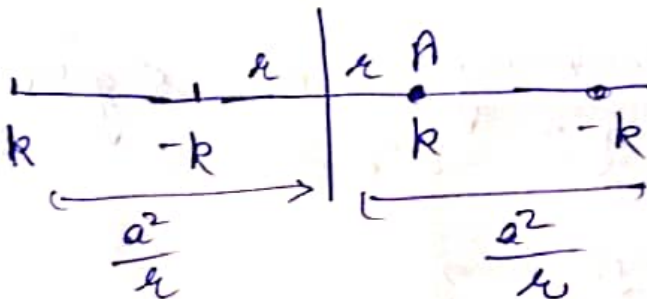
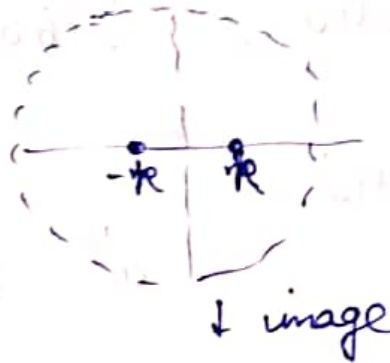
In 2nd complement

$$\begin{array}{r} \overline{1111} \\ + \quad 1 \\ \hline \end{array} = \begin{array}{r} 0000 \\ \quad 1 \\ \hline \end{array}$$

$$= (0001)_2 = (1)_{10}$$

- (c) A vortex pair is situated within a cylinder. Show that it will remain at rest if the distance of either from the centre is given by $(\sqrt{5} - 2)^{\frac{1}{2}} a$, where a is the radius of the cylinder.

(15)



$$\omega_{\text{total}} = \frac{ik}{2\pi} \log \frac{(z-k)(z+\frac{a^2}{k})}{(z+k)(z-\frac{a^2}{k})}$$

ω at A due to others

$$\omega = \frac{ik}{2\pi} \left[\log \left(z + \frac{a^2}{k} \right) - \log z - \frac{a^2}{k} - \log(z+k) \right]$$

$$\frac{d\omega}{dz} = \frac{ik}{2\pi} \left[\frac{1}{z + \frac{a^2}{k}} - \frac{1}{z - \frac{a^2}{k}} - \frac{1}{z+k} \right]$$

$$\text{at } z = k \quad \frac{d\omega}{dz} = 0$$

$$\frac{k}{k^2 + a^2} - \frac{k}{k^2 - a^2} - \frac{1}{2k} = 0$$

$$\frac{-2a^2 k}{(k^2 + a^2)(k^2 - a^2)} - \frac{1}{2k} = 0$$

$$4a^2x^2 + x^4 - a^4 = 0$$

$$x^2 = \frac{-4a^2 \pm \sqrt{16a^4 + 4a^4}}{2}$$

~~$$x^2 = \frac{-4a^2 \pm \sqrt{16a^4 + 4a^4}}{2}$$~~

$$x_1 x_2 = -a^4 (< 0)$$

$$x^2 > 0$$

$$\Rightarrow x^2 = \frac{-2a^2 + \sqrt{5}a^2}{2}$$

taking +ve sign

$$\frac{x^2}{a^2} = \sqrt{5} - 2$$

$$x = a(\sqrt{5} - 2)^{1/2}$$