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NEXT IAS

(To be filled by candidate)

Name of Candidate : VAIBHAV PANDEY

Roll No. : OAC24MTTN004 Contact No...

e-mail:... Date of Examination : 29/7/24

Exam Centre : Old Rajinder Nagar ☐ Online ☑

CSE (MAINS) TEST SERIES-2024

Test-05

Dated : 21-07-2024

MATHEMATICS OPTIONAL

Time Allowed : Three Hours

Maximum Marks: 250

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

*There are **EIGHT** questions divided in **Two sections** and printed in **ENGLISH**.*

*Candidate has to attempt **FIVE** questions in all.*

***Question No. 1 and 5** are compulsory and out of the remaining, any **THREE** are to be attempted choosing at least **ONE** question from each section.*

The number of marks carried by a question/part is indicated against it.

Answers must be written in the medium authorized in the Admission. Certificate which must be stated clearly on the cover of this Question-cum- Answer (QCA) booklet in the space provided.

No marks will be given for answers written in a medium other than the authorized one.

Word limit in questions, wherever specified, should be adhered to.

Attempts of question shall be counted in sequential order, unless struck off. Attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question cum Answer (QCA) booklet must be clearly struck off.

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(For filling by Examiners only)

Q. No.	Page No.	Max. Marks	Marks	Total	Signature
1. (a)	4	10			
1. (b)	6	10			
1. (c)	8	10			
1. (d)	10	10			
1. (e)	12	10			
2. (a)	14	20			
2. (b)	18	15			
2. (c)	21	15			
3. (a)	24	15			
3. (b)	27	15			
3. (c)	30	20			
4. (a)	34	20			
4. (b)	38	15			
4. (c)	41	15			
5. (a)	44	10			
5. (b)	46	10			
5. (c)	48	10			
5. (d)	50	10			
5. (e)	52	10			
6. (a)	54	15			
6. (b)	57	20			
6. (c)	61	15			
7. (a)	64	20			
7. (b)	68	15			
7. (c)	71	15			
8. (a)	74	20			
8. (b)	78	15			
8. (c)	81	15			
Grand Total					

Remarks

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Observations:

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SECTION-A

Q.1 (a) Let U and W be subspaces of vector space V and $x, y \in V$. Then prove that $x+U \subseteq y+W$ iff $U \subseteq W$ and $x-y \in W$. **(10)**

Let $U \subseteq W$, ~~and~~ $x-y \in W$ and $x+u \in x+U$

$\Rightarrow x+u = (x-y)+y+u$ {here $u \in U$}

$= y+(x-y)+u$ {$\because$ V is commutative}

Now, $x-y \in W$ and $u \in U \subseteq W$

$\Rightarrow (x-y)+u \in W$ {closure in W}

$\Rightarrow x+u = y+(x-y)+u \in y+W$

$\therefore x+U \subseteq y+W$

Now, let $x+U \subseteq y+W$

$\Rightarrow \forall u \in U$, $\exists w \in W$ s.t.

$x+u = y+w$

$\Rightarrow (x-y)+u = w \quad \forall u \in U$

$\Rightarrow (x-y)+u \in W \quad \forall u \in U$

Also, $0 \in U$ {$\because$ U is a vector space}

$\Rightarrow (x-y)+0 \in W$

$\Rightarrow x-y \in W$

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Now, $x-y+u \in W \quad \forall u \in U$

And, $x-y \in W$

$$(x-y)-(x-y)+u \in W-(x-y) \quad \forall u \in U$$

$\Rightarrow u \in W \quad \forall u \in U \quad \{\because x-y \in W\}$

$\therefore U \subseteq W.$

$\therefore U \subseteq W$ & $xy \in W$.

Hence, proved

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Q.1 (b) Show that 0 is characteristic root of matrix iff the matrix is singular. (10)

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Let 0 is characteristic root of a matrix A

$\Rightarrow |A - 0I| = 0$

$\Rightarrow |A| = 0$

$\Rightarrow$ A is singular.

Let A is singular

$\Rightarrow |A| = 0$

$\Rightarrow |A - 0 \cdot I| = 0$

$\Rightarrow$ 0 is characteristic root of A.

Hence, proved.

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Q.1 (c) Evaluate $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$ as $x \to 0$. **(10)**

Let $l = \lim_{x\to 0} (\cos x)^{1/x^2}$

$\Rightarrow \log l = \lim_{x\to 0} \log\left((\cos x)^{1/x^2}\right)$

{$\because \log x$ is continuous in its domain}

$\Rightarrow \log l = \lim_{x\to 0} \frac{\log(\cos x)}{x^2}$ { $\frac{0}{0}$ form }

$= \lim_{x\to 0} \frac{\frac{1}{\cos x}(-\sin x)}{2x}$ { L' Hospital rule }

$= \lim_{x\to 0} \frac{-1}{2} \frac{\tan x}{x}$

$= \frac{-1}{2}$ { $\because \lim_{x\to 0} \frac{\tan x}{x} = 1$ }

$\Rightarrow l = e^{-1/2}$

$\therefore \boxed{\lim_{x\to 0} (\cos x)^{1/x^2} = \frac{1}{\sqrt{e}}}$

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Q.1 (d) Test the convergence of $\int_0^1 \frac{\sin\left(\frac{1}{x}\right)}{\sqrt{x}}dx$ **(10)**

$x=0$ is the only point of infinite discontinuity.

At $x=1$: $\lim_{x\to 1^-} \frac{\sin\left(\frac{1}{x}\right)}{\sqrt{x}} = \sin 1$

which is finite.

Now, let $f(x) = \frac{\sin\frac{1}{x}}{\sqrt{x}}$

$\Rightarrow |f(x)| = \frac{\left|\sin\frac{1}{x}\right|}{\sqrt{x}} \le \frac{1}{\sqrt{x}}$

$\left\{\because \left|\sin\frac{1}{x}\right| \le 1\right\}$

Now, $\int_0^1 |f(x)|\,dx \le \int_0^1 \frac{1}{\sqrt{x}}dx$

$\downarrow$

This is convergent.

$\therefore \int_0^1 |f(x)|\,dx$ is convergent.

$\therefore \int_0^1 f(x)\,dx$ is convergent.

(Absolute convergence $\Rightarrow$ convergence)

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Q.1 (e) Find the equation of the cylinder whose generators touch the sphere $x^2+y^2+z^2=9$ and are perpendicular to the plane $x-y-3z=5$. **(10)**

The generators have direction ratios $(1,-1,-3)$.

Let (x_1, y_1, z_1) be a point on the cylinder.

$\therefore$ Equation of generator through (x_1, y_1, z_1) is

$$\frac{x-x_1}{1} = \frac{y-y_1}{-1} = \frac{z-z_1}{-3}$$

A general point on this line is

(x_1+r, y_1-r, z_1-3r) where r is a parameter.

Putting in equation of sphere,

$$(x_1+r)^2 + (y_1-r)^2 + (z_1-3r)^2 = 9$$

$$11r^2 + (2x_1 - 2y_1 - 6z_1)r + (x_1^2+y_1^2+z_1^2-9) = 0$$

$\therefore$ the generator touches the sphere, above quadratic should have equal roots

$$\Rightarrow (2x_1 - 2y_1 - 6z_1)^2 - 4\times 11 (x_1^2+y_1^2+z_1^2-9) = 0$$

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$$x_1^2 + y_1^2 + 9z_1^2 - 2x_1y_1 + 6y_1z_1 - 6x_1z_1 - 11x_1^2 - 11y_1^2 - 11z_1^2 + 99 = 0$$

$$\Rightarrow \boxed{10x^2 + 10y^2 + 2z^2 + 2xy - 6yz + 6xz - 99 = 0}$$

is the equation of the required cylinder.



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Q.2 (a) Find non-singular matrices P, Q so that PAQ is a normal form where $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ and hence find its rank. (20)

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix} = I_3 A I_4$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 3 & -3 & 1 & 2 \\ 2 & 1 & -3 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -6 & -2 & -4 \\ 0 & -1 & -5 & -10 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, $C_4 \to C_4 - 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & -2 & -4 \\ 0 & -1 & -5 & -10 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$R_2 \leftrightarrow R_3$ and $R_2 \to -R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 10 \\ 0 & -6 & -2 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 + 6R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 28 & 56 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ -6 & 1 & 9 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 - 5C_2$, $C_4 \to C_4 - 10C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 28 & 56 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ -6 & 1 & 9 \end{bmatrix} A \begin{bmatrix} 1 & -1 & 4 & 8 \\ 0 & 1 & -5 & -10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to \frac{R_3}{28}$;

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ \frac{-3}{14} & \frac{1}{28} & \frac{9}{28} \end{bmatrix} A \begin{bmatrix} 1 & -1 & 4 & 8 \\ 0 & 1 & -5 & -10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_4 \to C_4 - 2C_3$

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$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ -\frac{3}{14} & \frac{1}{28} & \frac{9}{28} \end{bmatrix} A \begin{bmatrix} 1 & -1 & 4 & -2/7 \\ 0 & 1 & -5 & -1/7 \\ 0 & 0 & 1 & -11/7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ -\frac{3}{14} & \frac{1}{28} & \frac{9}{28} \end{bmatrix} A \begin{bmatrix} 1 & -1 & 4 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

↳ This is in normal form.

$$\therefore P = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ -\frac{3}{14} & \frac{1}{28} & \frac{9}{28} \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & -1 & 4 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $PAQ = \begin{bmatrix} I_3 & 0 \end{bmatrix}$

$\therefore$ Rank of $A = 3$

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Q.2 (b) Find the whole area included between the curve $x^2y^2 = a^2(y^2 - x^2)$ and its asymptotes. **(15)**

$x^2y^2 = a^2(y^2 - x^2)$

L.H.S. is ≥ 0

$\therefore$ curve exists only for $|y| > |x|$

Also, it is symmetric in all 4 quadrants.

Now, in 1st quadrant :-

$x^2y^2 = a^2(y^2 - x^2)$

$\Rightarrow x^2y^2 = a^2y^2 - a^2x^2$

$\Rightarrow y^2(a^2 - x^2) = a^2x^2$

$\Rightarrow y = \dfrac{ax}{\sqrt{a^2 - x^2}}$ $\Rightarrow x < |a|$

$$\frac{dy}{dx} = \frac{\left(\sqrt{a^2-x^2}\right)(a) - (ax)\cdot\dfrac{(-2x)}{2\sqrt{a^2-x^2}}}{(a^2-x^2)}$$

$$= \frac{(a^2-x^2)a + ax^2}{(a^2-x^2)^{3/2}}$$

$$= \frac{a^3}{(a^2-x^2)^{3/2}}$$

For $x < a$, $\dfrac{dy}{dx} > 0$ $\therefore$ $y(x)$ is increasing in 1st quadrant

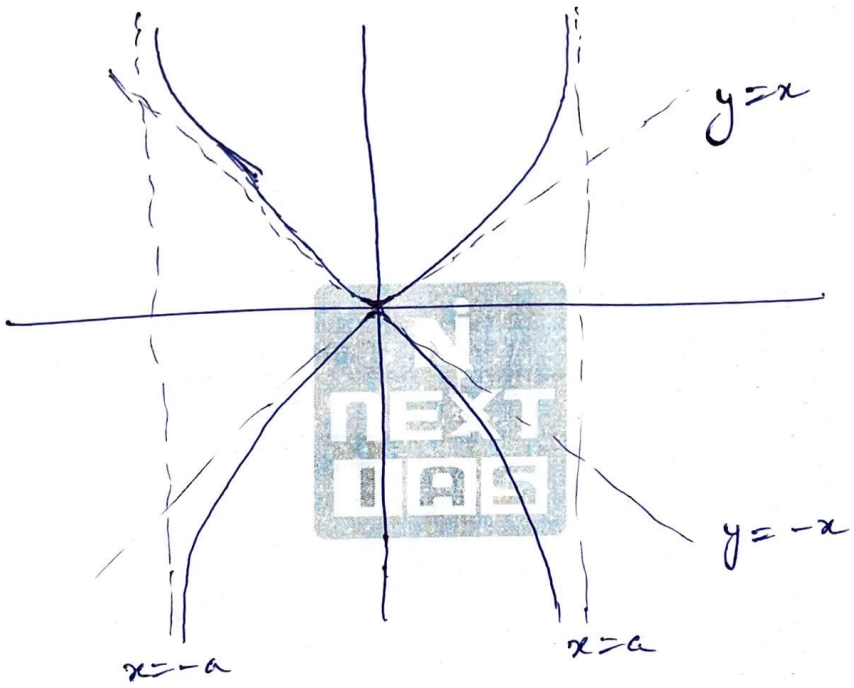
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$$\left.\frac{dy}{dx}\right|_{x=0} = 1$$

Also, $y \to \infty$ as $x \to a$

$$\frac{dy}{dx} \to \infty \quad \text{as} \quad x \to a$$



The asymptotes are thus, $\boxed{x = \pm a}$

$$\text{Required area} = 4\int_{x=0}^{a} \frac{ax}{\sqrt{a^2-x^2}}\,dx$$

Put $x = a\sin\theta$

$\Rightarrow dx = a\cos\theta\,d\theta$

$$\therefore \text{Area} = 4\int_{\theta=0}^{\pi/2} \frac{a^2\sin\theta \cdot a\cos\theta\,d\theta}{a\cos\theta}$$

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$\Rightarrow$ Area $= 4a^2$

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Q.2 (c) Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ through the point (α,β,γ). Prove that the perpendiculars to them through the origin generate the cone $(\alpha x+\beta y+\gamma z)^2=a^2x^2+b^2y^2+c^2z^2$. (15)

Let a perpendicular through origin described above be

$$\frac{x}{l}=\frac{y}{m}=\frac{z}{n} \quad \rightarrow (1)$$

Now, tangent plane to ellipsoid perpendicular to this line is

$$lx+my+nz=\pm\sqrt{a^2l^2+b^2m^2+c^2n^2}$$

{ condition for tangency on central conicoid }

But it passes through (α,β,γ)

$$\Rightarrow \alpha l+\beta m+\gamma n=\pm\sqrt{a^2l^2+b^2m^2+c^2n^2}$$

$$\Rightarrow (\alpha l+\beta m+\gamma n)^2=a^2l^2+b^2m^2+c^2n^2 \quad \rightarrow (2)$$

From (1) & (2), the given generators through origin generate the cone

$$(\alpha x+\beta y+\gamma z)^2=a^2x^2+b^2y^2+c^2z^2.$$

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as the relation in l, m, n in equation (2) is homogeneous in l, m, n.



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Q.3 (a) For what values of λ and μ, the system of equations $\begin{aligned} x+y+z&=6 \\ x+2y+3z&=10 \\ x+2y+\lambda z&=\mu \end{aligned}$ has

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions (15)



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Q.3 (b) Find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ using Lagrange's method of multipliers. **(15)**



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Q.3 (c) Prove that $5x^2+5y^2+8z^2+8yz+8zx-2xy+12x-12y+6=0$ represents a cylinder whose cross section is an ellipse of eccentricity $1/\sqrt{2}$. (20)



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Q.4 (a) Show that the set $\{(1,2,0,3),(4,0,5,8),(8,1,5,6)\}$ is linearly independent in IR^4. The vectors form a basis for a three-dimensional subspace V of IR^4. Construct an orthonormal basis for V. **(20)**



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Q.4 (b) Find the asymptotes of $x^3+2x^2y-xy^2-2y^3+xy-y^2-1=0$. (15)



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Q.4 (c) Prove that the centres of the spheres which touch the lines $y=mx,\ z=c$ and $y=-mx, z=-c$ must lie on the conicoid $mxy+c\left(1+m^2\right)z=0.$ (15)

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SECTION-B

Q.5 (a) Find the Wronskian of the set of functions $\{3x^3, |x^3|\}$ on the interval $[-1,1]$ and determine whether the set is linearly dependent on $[-1,1]$. **(10)**

Let $f_1(x) = 3x^3$, $f_2(x) = |x^3|$

for $f_2(x)$ at $x=0$

R.H.D. $= \lim_{h\to 0} \frac{h^3 - 0}{h} = 0$

L.H.D. $= \lim_{h\to 0} \frac{-(-h)^3 - 0}{-h} = 0$

$\therefore$ $f_2(x)$ is differentiable on $[-1,1]$.

$f_1(x)$ is also differentiable on $[-1,1]$.

Now, Wronskian $W(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ f_1'(x) & f_2'(x) \end{vmatrix}$

For $1 \geq x \geq 0$

$$W(x) = \begin{vmatrix} 3x^3 & x^3 \\ 9x^2 & 3x^2 \end{vmatrix} = 0$$

for $-1 \leq x < 0$

$$W(x) = \begin{vmatrix} 3x^3 & -x^3 \\ 9x^2 & -3x^2 \end{vmatrix} = 0$$

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$\therefore$ $W(x)$ is identically zero on $[-1, 1]$.

$\therefore$ $\{f_1(x), f_2(x)\}$ is linearly dependent on $[-1, 1]$.



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Q.5 (b) The necessary and sufficient condition for $f(t)$ to be constant is $\frac{df}{dt}=0$. (10)

Let $\vec{f}(t)$ be a constant vector function of scalar variable t.

$\therefore \vec{f}(t) = f_1(t)\hat{i} + f_2(t)\hat{j} + f_3(t)\hat{k} = \vec{c}$

where $\vec{c}$ is a constant vector

$\Rightarrow \frac{d\vec{f}}{dt} = \frac{df_1(t)}{dt}\hat{i} + \frac{df_2(t)}{dt}\hat{j} + \frac{df_3(t)}{dt}\hat{k}$

$= \frac{dc_1}{dt}\hat{i} + \frac{dc_2}{dt}\hat{j} + \frac{dc_3}{dt}\hat{k} = \frac{d\vec{c}}{dt}$

$\Rightarrow \frac{d\vec{f}}{dt} = 0\hat{i} + 0\hat{j} + 0\hat{k}$ $\left\{\because f_1(t), f_2(t), f_3(t) \text{ are constant & } c_1, c_2, c_3 \text{ are constant.}\right\}$

$\Rightarrow \frac{d\vec{f}}{dt} = \vec{0}$.

Now, let $\frac{d\vec{f}}{dt} = \vec{0}$

$\Rightarrow \frac{df_1(t)}{dt}\hat{i} + \frac{df_2(t)}{dt}\hat{j} + \frac{df_3(t)}{dt}\hat{k} =$

$0\hat{i} + 0\hat{j} + 0\hat{k}$

$\Rightarrow \frac{df_1(t)}{dt} = 0$, $\frac{df_2(t)}{dt} = 0$, $\frac{df_3(t)}{dt} = 0$

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~~$\Rightarrow f_1 = c_1 \;,\; f_2 = c_2$~~

$\Rightarrow f_1(t) = c_1 \;,\; f_2(t) = c_2 \;,\; f_3(t) = c_3$

where c_1, c_2, c_3 are constant scalars

$\Rightarrow \vec{f}(t) = f_1(t)\,\hat{i} + f_2(t)\,\hat{j} + f_3\,\hat{k}$

$= \vec{c}$ $\quad \left\{ \text{where } \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k} \right\}$

$\therefore \vec{f}(t)$ is a constant vector.

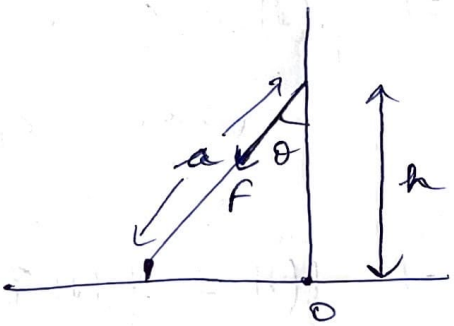


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Q.5 (c) At what point of a tree must one of a rope of given length 'a' be attached so that a man pulling at the other end may exert greatest force to pull it over? **(10)**

Let the force applied by man be f and rope be attached at height h.



The tree has a tendency to rotate about point O upon being pulled over.

$\therefore$ Taking Torque about O :-

$$M. = (F \sin\theta)\, h$$

We want maximum value of M.

i.e. $$M = \left(F \frac{\sqrt{a^2-h^2}}{a} \right) h$$

Let $f(h) = h\sqrt{a^2-h^2}$

$\Rightarrow f^2(h) = h^2(a^2-h^2)$

$$\therefore \frac{d(f^2(h))}{d(h^2)} = h^2(-1) + (a^2-h^2)(1)$$

$$\therefore \frac{df^2(h)}{d(h^2)} = 0$$

$\Rightarrow 2h^2 = a^2$

$\Rightarrow h = \frac{a}{\sqrt{2}}$

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Also, $\dfrac{d^2\left(f^2(h)\right)}{\left(d(h^2)\right)^2} = -2 < 0$

$\therefore$ At $h = \dfrac{a}{\sqrt{2}}$, we have maxima.

$\therefore$ The rope should be attached at height $\boxed{h = \dfrac{a}{\sqrt{2}}}$.

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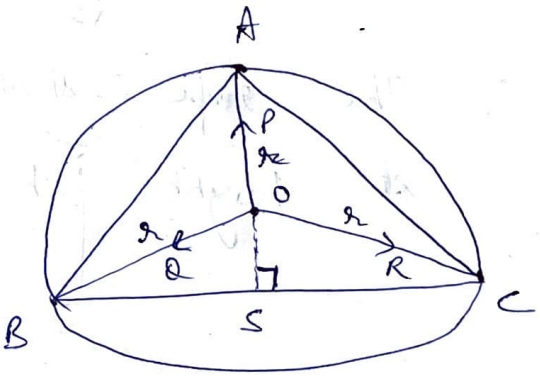
Q.5 (d) A particle is placed at the centre of a circle and is acted on by three forces P, Q and R tending towards the angular points A, B and C of a circumscribed triangle. Prove that when the forces are in equilibrium then $\frac{P}{\cos\frac{A}{2}} = \frac{Q}{\cos\frac{B}{2}} = \frac{R}{\cos\frac{C}{2}}$. (10)

~~$\triangle BOS$ and $\triangle COS$ are congruent~~

$\therefore \angle BOS = \angle COS$

~~Also,~~ $\angle BOC = 2A$

~~$\Rightarrow \angle BOS = \angle COS = A$~~



Similarly, $\angle AOC = 2B$, $\angle AOB = 2C$

By Lami's theorem :-

$$\frac{P}{\sin 2A} = \frac{Q}{\sin 2B} = \frac{R}{\sin 2C}$$

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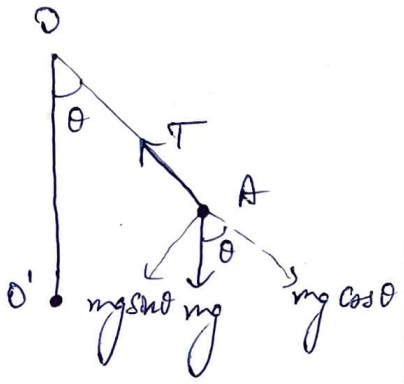
Q.5 (e) One end of the inextensible string is fixed to a point O and to the other end is tied a particle of mass m. The particle is projected from its position of equilibrium vertically below O with horizontal velocity so as to carry it right round the circle. Prove that the sum of the tensions at the ends of a diameter is constant. **(10)**

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Writing $\bar{F} = ma$ in radial direction

$$T - mg\cos\theta = \frac{mv^2}{r}$$

$$\Rightarrow T = \frac{m}{r}\left(gr\cos\theta + v^2\right) \quad -(1)$$



At point A of the path, angle subtended by OA with vertical is θ.

At opposite end of diameter, angle subtended is $\pi + \theta$.

By conservation of Energy at two diametrically opposite points :-

$$\frac{1}{2}mv_1^2 + mgr(1-\cos\theta) = \frac{1}{2}mv_2^2 + mgr(1-\cos(\pi+\theta))$$

If T_1 and T_2 are tensions at diametrically opposite points,

$$T_1 + T_2 = \frac{m}{r}\left(gr\cos\theta + v_1^2 + gr\cos(\pi+\theta) + v_2^2\right)$$

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$\Rightarrow T_1 + T_2 = \frac{m}{r}(v_1^2 + v_2^2)$

Now, applying Conservation of energy between lowest point of & for angle θ :-

$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$

{Let initial velocity = u}

$\Rightarrow v^2 = u^2 - 2gr(1-\cos\theta)$

$\therefore T_1 + T_2 = \frac{m}{r}(v_1^2 + v_2^2)$

$= \frac{m}{r}\left(u^2 - 2gr(1-\cos\theta) + u^2 - 2gr(1-\cos(\pi+\theta))\right)$

$= \frac{m}{r}(2u^2 - 4gr)$

which is independent of θ.

$\therefore$ sum of tensions at the ends of a diameter is constant.

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Q.6 (a) Find the general solution of $ayp^2+(2x-b)p-y=0, a>0.$ (15)

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Q.6 (b) Find the law to force of the pole when path of a particle is the cardioid $r = a(1-\cos\theta)$ and prove that if F be the force at the apse and v the velocity there, then $3v^2 = 4aF$. (20)

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Q.6 (c) Evaluate $\iint_S curl\, A.\, dS$, where S is the open surface $x^2+y^2-4x+4z=0, z\geq 0$ and $A=\left(y^2+z^2-x^2\right)\hat{i}+\left(2z^2+x^2-y^2\right)\hat{j}+\left(x^2+y^2-3z^2\right)\hat{k}$. **(15)**



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Q.7 (a) Solve $(x+2)\frac{d^2y}{dx^2}-(2x+5)\frac{dy}{dx}+2y=(x+1)e^x$. (20)

The homogeneous part of the given D.E. is

$$(x+2)\frac{d^2y}{dx^2}-(2x+5)\frac{dy}{dx}+2y=0$$

for $u(x)=e^{2x}$, above D.E. is satisfied

Converting given D.E. to standard form,

$$\frac{d^2y}{dx^2}-\underbrace{\frac{(2x+5)}{(x+2)}}_{P}\frac{dy}{dx}+\underbrace{\frac{2}{(x+2)}}_{Q}y=\underbrace{\frac{(x+1)e^x}{(x+2)}}_{R}$$

Let $y=u(x)\,v(x)$ be the complete solution of given D.E.

Changing dependent variable where a solution to homogeneous part is known,

$$P+\frac{2}{u}\frac{du}{dx}=\frac{-(2x+5)}{x+2}+\frac{2}{e^{2x}}\cdot 2e^{2x}$$

$$=\frac{-2x-5+4x+8}{x+2}$$

$$=\frac{2x+3}{x+2}$$

$\therefore$ D.E. on $v(x)$ is

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$$\frac{d^2v}{dx^2} + \left(\frac{2x+3}{x+2}\right)\frac{dv}{dx} = \frac{(x+1)}{(x+2)e^x}$$

I.F. for above linear D.E. ↑

$$e^{\int \frac{2x+3}{x+2}\,dx} = e^{2x - \ln(x+2)}$$

$$= \frac{e^{2x}}{(x+2)}$$

$$\therefore \frac{e^{2x}}{(x+2)}\frac{dv}{dx} = \int \frac{(x+1)}{(x+2)^2} e^x\,dx$$

$$= \int \frac{e^x\,dx}{(x+2)} - \int \frac{e^x}{(x+2)^2}\,dx$$

$$= \frac{e^x}{x+2} + \int \frac{e^x}{(x+2)^2}\,dx - \int \frac{e^x\,dx}{(x+2)^2}$$

$$= \frac{e^x}{x+2} + c_1$$

$$\Rightarrow \frac{dv}{dx} = e^{-x} + c_1\frac{(x+2)}{e^{2x}}$$

$$\Rightarrow v = -e^{-x} + c_1\int (x+2)e^{-2x}\,dx + c_2$$

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$$\Rightarrow v = -e^{-x} + C_1\left\{\frac{(x+2)e^{-2x}}{-2} - \int\frac{e^{-2x}}{-2}dx\right\} + C_2$$

$$\Rightarrow v = -e^{-x} + C_1\left\{-\frac{(x+2)e^{-2x}}{2} - \frac{e^{-2x}}{4}\right\} + C_2$$

$$\Rightarrow v = -e^{-x} + C_1'\left\{(x+2)e^{-2x} + \frac{e^{-2x}}{2}\right\} + C_2$$

$$\Rightarrow v = -e^{-x} + C_1''(2x+5)e^{-2x} + C_2$$

$$\therefore \boxed{y = uv = -e^{x} + C_1''(2x+5) + C_2e^{2x}}$$

is the complete solution.

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Q.7 (b) Suppose $A=(3x^2+6y)\hat{i}-14yz\hat{j}+20xz^2\hat{k}$. Evaluate $\int_c A\,dr$ from $(0,0,0)$ to $(1,1,1)$ along the following paths C:

(i) $x=t, y=t^2, z=t^3$.

(ii) The straight lines from $(0,0,0)$ *to* $(1,0,0)$, then to $(1,1,0)$ and then to $(1,1,1)$

(iii) The straight line joining $(0,0,0)$ and $(1,1,1)$. (15)

(i) $$\int_c \bar{A}\cdot d\bar{r} = \int_c \left(\bar{A}\cdot\frac{d\bar{r}}{dt}\right)dt$$

$$= \int_c \left\{(3t^2+6t^2)(1) - 14t^5(2t) + 20t^7(3t^2)\right\} dt$$

$$= \int_{t=0}^{1} \left(3t^2+6t^2-28t^6+60t^9\right)dt$$

$$= 1+2-4+6$$

$$= \boxed{5}$$

(ii) Let C_1: $(t,0,0)$ $0\le t\le 1$

C_2: $(1,t,0)$ $0\le t\le 1$

C_3: $(1,1,t)$ $0\le t\le 1$

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$$I = \int_{C_1} \vec{A}\cdot d\vec{r} + \int_{C_2} \vec{A}\cdot d\vec{r} + \int_{C_3} \vec{A}\cdot d\vec{r}$$

$$= \int_{t=0}^{1} 3t^2\,dt + \int_{t=0}^{1} 0\,dt + \int_{t=0}^{1} 20t^2\,dt$$

$$= 1 + 0 + \frac{20}{3}$$

$$= \boxed{\frac{23}{3}}$$

(iv) $C: t\hat{i} + t\hat{j} + t\hat{k}$ where $0 \le t \le 1$

$$\therefore I = \int \vec{A}\cdot d\vec{r}$$

$$= \int_{t=0}^{1} \left(3t^2 + 6t - 14t^2 + 20t^3\right) dt$$

$$= 3 - \frac{11}{3} + 5$$

$$= \boxed{\frac{13}{3}}$$

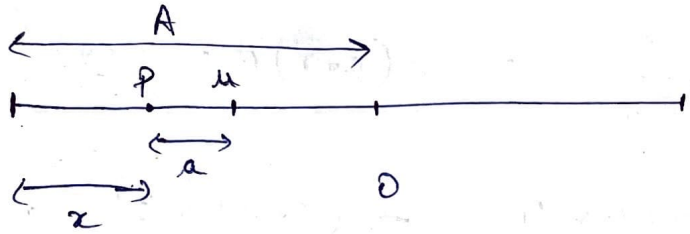
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Q.7 (c) If in a simple harmonic motion u,v,w be the velocities at a distance a,b,c from a fixed point on the straight line (which is not the centre of force), show that the period T is given by the equation $\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b)=\begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$. (15)



Let O be mean position, P the point on the line from which distances are measured and A the amplitude of SHM.

In an SHM, velocity at distance x from mean position is

$$V = \frac{2\pi}{T}\sqrt{A^2 - x^2}$$

{ T → Time period, A → Amplitude }

For our case then, for the first point, we have

$$u^2 = \frac{4\pi^2}{T^2}\left(A^2 - (A-(a+x))^2\right)$$

$$\Rightarrow \frac{u^2 \cdot T^2}{4\pi^2} = (a+x)\left(2A - (a+x)\right)$$

$$\Rightarrow \frac{u^2 T^2}{4\pi^2} = 2(a+x)A - (a+x)^2 \quad \text{---} \quad (1)$$

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Similarly, for the other two points,

$$v^2\frac{T^2}{4\pi^2} = 2(b+x)A - (b+x)^2 \quad \text{— (2)}$$

And, $$w^2\cdot\frac{T^2}{4\pi^2} = 2(c+x)A - (c+x)^2 \quad \text{— (3)}$$

Now, $(b+x)\times(1) - (a+x)\times(2)$ gives,

$$\frac{T^2}{4\pi^2}\left(u^2(b+x) - v^2(a+x)\right) = (a+x)(b+x)(b-a)$$

Similarly, $(c+x)\times(2) - (b+x)\times(3)$ gives,

$$\frac{T^2}{4\pi^2}\left(v^2(c+x) - w^2(b+x)\right) = (c+x)(b+x)(c-b)$$

And $(a+x)\times(3) - (c+x)\times(1)$ gives,

$$\frac{T^2}{4\pi^2}\left(w^2(a+x) - u^2(c+x)\right) = (c+x)(a+x)(a-c)$$

Adding above three,

$$\frac{T^2}{4\pi^2}\left(u^2(b-c) + v^2(c-a) + w^2(a-b)\right)$$

$$= (a+x)(b+x)(c+x)\left\{\frac{b-a}{c+x} + \frac{c-b}{a+x} + \frac{a-c}{b+x}\right\}$$

$$= (a-b)(b-c)(c-a)$$

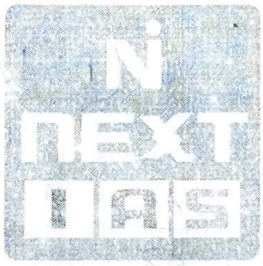
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$$\Rightarrow \frac{4\pi^2}{T^2}(a-b)(b-c)(c-a)$$

$$= \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

Hence proved.



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Q.8 (a) Solve by the method of variation of parameters the differential equation $x''(t)-2\frac{x(t)}{t^2}=t, where 0<t<\infty$. **(20)**

First solving the homogenious part for Complementary function

$$x''(t) - \frac{2x(t)}{t^2} = 0$$

$$\Rightarrow t^2 x''(t) - 2(x(t)) = 0$$

Let $t = e^z$ { Cauchy – Euler D.E. }

$$\Rightarrow t^2 \frac{d^2x}{dt^2} = D_1(D_1-1)x$$

where $D_1 = \frac{d}{dz}$

$\therefore$ D.E becomes

$$(D_1(D_1-1) - 2)x = 0$$

$$\Rightarrow (D_1^2 - D_1 - 2)x = 0$$

Auxillary equation is

$$m^2 - m - 2 = 0$$

$$\Rightarrow m = \frac{1 \pm 3}{2} = -1, 2$$

$$\therefore x = c_1 e^{-z} + c_2 e^{2z}$$

$$\Rightarrow x(t) = c_1 e^{-\log t} + c_2 e^{2\log t} \quad \{\because t = e^z\}$$

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$$\Rightarrow x(t) = \frac{c_1}{t} + c_2 t^2$$

$\therefore$ Complementary function for original D.E.

& C.F. $\equiv c_1 \cdot \frac{1}{t} + c_2 t^2$

Now, to find P.I.

$u(t) = \frac{1}{t}$, $v(t) = t^2$, $R(t) = t$

Let P.I. be $A(t)\,u(t) + B(t)\,v(t)$

$$uv' - vu' = \begin{vmatrix} \frac{1}{t} & t^2 \\ \frac{-1}{t^2} & 2t \end{vmatrix} = 3$$

$$\therefore A(t) = \int \frac{-vR}{uv' - vu'}$$

$$= -\int \frac{t^2 \cdot t}{3}\,dt$$

$$= \frac{-t^4}{12}$$

And, $B(t) = \int \frac{uR}{uv' - vu'}$

$$= \int \frac{1}{3}\,dt$$

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$$= \frac{t}{3}$$

$\therefore$ P.I. is $Au + Bv$

$$= -\frac{t^3}{12} + \frac{t^3}{3}$$

$$= \frac{t^3}{4}$$

$\therefore$ general solution of given D.E. is

$$x(t) = C.F. + P.I.$$

$$\Rightarrow \boxed{x(t) = \frac{C_1}{t} + C_2 t^2 + \frac{t^3}{4}.}$$

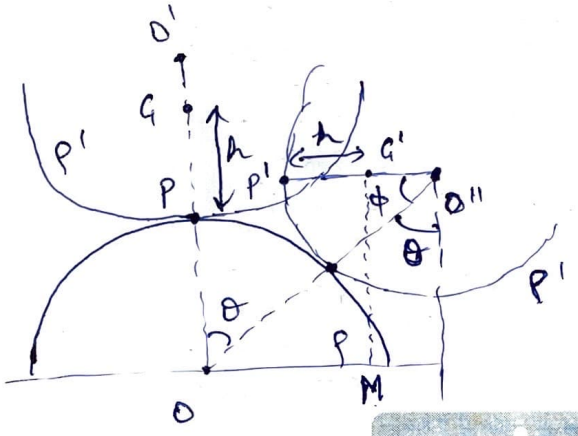
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Q.8 (b) Suppose a cylinder of any cross section is balanced on another fixed cylinder, the contact of curved surface being rough and the common tangent line horizontal. Let ρ and ρ' be radii of curvature of the two cylinders at the point of contact and h be the height of centre of gravity of the upper cylinder above the point of contact. Show that the upper cylinder is balanced in stable equilibrium if $h < \frac{\rho\rho'}{\rho+\rho'}$. (15)



G is centre of gravity initially & G' afterwards

Initial height of centre of gravity above O is $OG = (\rho + h)$

After upper cylinder has rotated by an angle ϕ then height of Centre of gravity is MG'

$$MG' = OO''\cos\theta - O''G'\cos(\theta+\phi)$$

$$\Rightarrow z = (\rho+\rho')\cos\theta - (\rho'-h)\cos(\theta+\phi)$$

Also, as no slipping happening between the two cylinders $\Rightarrow \rho\theta = \rho'\phi$

$$\Rightarrow \frac{d\theta}{d\phi} = \frac{\rho'}{\rho} \quad -(1)$$

Now, $z = (\rho+\rho')\cos\theta - (\rho'-h)\cos\left(\theta + \frac{\rho}{\rho'}\theta\right)$

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$$\Rightarrow \frac{dz}{d\theta} = -(\rho+\rho')\sin\theta + (\rho'-h)\sin\left(\frac{\rho'+\rho}{\rho'}\theta\right)\left(\frac{\rho'+\rho}{\rho'}\right)$$

For $\theta = 0$, $\frac{dz}{d\theta} = 0$

$\therefore \theta = 0$ is position of equilibrium when tangent line at point of contact is horizontal.

Now, $\frac{d^2z}{d\theta^2} = -(\rho+\rho')\cos\theta + \frac{(\rho'-h)(\rho'+\rho)^2}{(\rho')^2}\cos\left(\frac{\rho'+\rho}{\rho'}\theta\right)$

At $\theta = 0$, $\frac{d^2z}{d\theta^2} = \frac{(\rho'+\rho)^2}{(\rho')^2}(\rho'-h) - (\rho+\rho')$

$$\Rightarrow \frac{d^2z}{d\theta^2} = (\rho'+\rho)\left(\frac{(\rho'+\rho)(\rho'-h) - (\rho')^2}{(\rho')^2}\right)$$

$$= \left(\frac{\rho'+\rho}{(\rho')^2}\right)\left(\rho\rho' - (\rho'+\rho)h\right)$$

Only force doing work in this situation is force of gravity.

$\therefore$ by z-test, for stable equilibrium

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$$\frac{d^2z}{d\theta^2} > 0$$

$$\Rightarrow pp' - (p'+p)h > 0$$

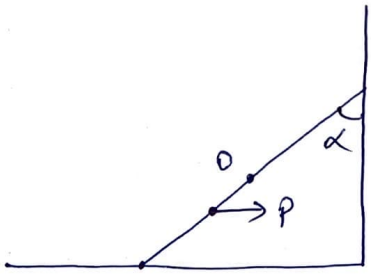
$$\Rightarrow \boxed{h < \frac{pp'}{p+p'}}$$

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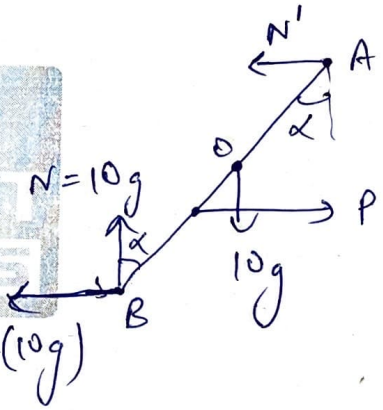
Q.8 (c) A uniform ladder of 10m length and of 10kg weight rests with its foot on the rough ground and its upper end against a smooth wall, the inclination to the vertical being α. A force P is applied horizontally to the ladder at a point distance 3m from the foot, so as to make the foot approach the wall. Prove that the force P must exceed $\frac{100}{7}\left(\mu+\frac{1}{2}\tan\alpha\right)$, where μ is the coefficient of friction at the foot. (15)

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At limiting point,
friction force $= \mu N$
where $N \equiv$ normal



Taking torque about point A,



$$(P\cos\alpha)(7) + (10g\sin\alpha)(5) = (10g\sin\alpha)(10) + (10\mu g\cos\alpha)(10)$$

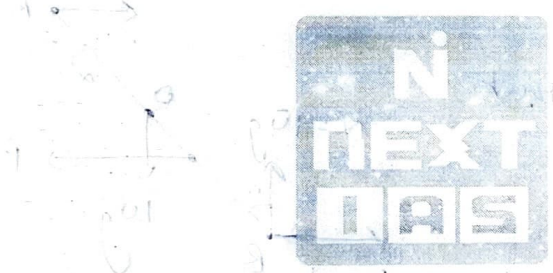
$$\Rightarrow 7P = \frac{50g\sin\alpha + 100\mu g\cos\alpha}{\cos\alpha}$$

$$\Rightarrow P = \frac{100}{7}g\left(\mu + \frac{1}{2}\tan\alpha\right)$$

$\therefore$ for ladder P to move, P must exceed $\frac{100}{7}\left(\mu+\frac{1}{2}\tan\alpha\right)$ kg weight.

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