

NEXT IAS

(To be filled by candidate)

Name of Candidate : Kamishak Aggarwal

Roll No. : MT 24MATL1011 Date of Examination : 31/7

Mobile Number : K mail nilom

Exam Centre : Old Rajinder Nagar ☐ Online ☒

CSE (MAINS) TEST SERIES-2024

Test-04	MATHEMATICS
Dated : 14-07-2024	OPTIONAL

Time Allowed : Three Hours

Maximum Marks: 250

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in Two sections and printed in ENGLISH.

Candidate has to attempt FIVE questions in all.

Question No. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each section.

The number of marks carried by a question/part is indicated against it.

Answers must be written in the medium authorized in the Admission Certificate which must be stated clearly on the cover of this Question cum Answer (QCA) booklet in the space provided.

No marks will be given for answers written in a medium other than the authorized one.

Word limit in questions, wherever specified, should be adhered to.

Attempts of question shall be counted in sequential order, unless struck off. Attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question cum Answer (QCA) booklet must be clearly struck off.

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(For filling by Examiners only)

Q. No.	Page No.	Max. Marks	Marks	Total	Signature
1. (a)	4	10			
1. (b)	6	10			
1. (c)	8	10			
1. (d)	10	10			
1. (e)	12	10			
2. (a)	14	20			
2. (b)	18	15			
2. (c)	21	15			
3. (a)	24	20			
3. (b)	27	15			
3. (c)	30	15			
4. (a)	34	15			
4. (b)	37	20			
4. (c)	40	15			
5. (a)	44	10			
5. (b)	46	10			
5. (c)	48	10			
5. (d)	50	10			
5. (e)	52	10			
6. (a)	54	15			
6. (b)	57	15			
6. (c)	60	20			
7. (a)	64	15			
7. (b)	67	15			
7. (c)	70	20			
8. (a)	74	20			
8. (b)	78	15			
8. (c)	81	15			
Grand Total					

Remarks

Observations:

Q.1 (b)

State the function $f(x, y, z) = x + \overline{xy} + \overline{xz} + z$ in disjunctive normal form (DNF)

for the function.

(10)

$$f(x, y, z) = x + \overline{xy} + \overline{xz} + z$$

DNF = Sum of products.

$$\begin{aligned} \overline{xy} + \overline{xz} &= \overline{xy} \cdot \overline{xz} \\ &= xy \cdot (\overline{x} + \overline{z}) \\ &= xy(x + \overline{z}) \\ &= xy + x\overline{y}\overline{z} = xy(1 + \overline{z}) \\ &= xy \end{aligned}$$

$$\begin{aligned} f(x, y, z) &= x + xy + z \\ &= x(1 + y) + z \\ &= x + z. \end{aligned}$$

$$\begin{aligned} &= x(y + \overline{y})(z + \overline{z}) + (x + \overline{x})(y + \overline{y}) \\ &= xy\overline{z} + x\overline{y}\overline{z} + x\overline{y}z + x\overline{y}\overline{z} + \overline{x}y\overline{z} \\ &\quad + \overline{x}\overline{y}z. \end{aligned}$$

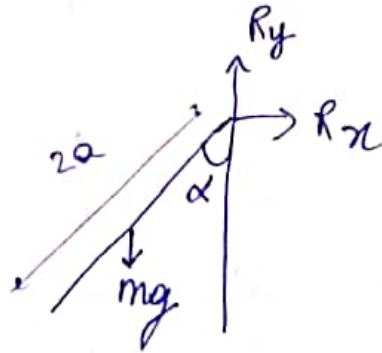
$$\frac{f}{z} = x + 3$$

x	y	z	f
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1

- Q.1 (d) A rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle α . Prove that the direction of reaction at the hinge makes with the vertical, an angle

$$\tan^{-1} \left[\frac{3}{4} \tan \alpha \right].$$

(10)



$$R_y = mg$$

$$R_x = m(2a)\omega^2$$

$$-R_x(a \cos \alpha) + R_y(a \sin \alpha) = \frac{m(2a)^2 \omega^2}{12}$$

$$R_y(a \sin \alpha) = \frac{m(2a)^2 \omega^2}{12}$$

$$-R_x(a \cos \alpha) + R_y(a \sin \alpha) = \frac{R_y a^2}{3g}$$

$$\omega = 0$$

$$\Rightarrow R_y \sin \alpha = R_x \cos \alpha$$

NEXT IAS

$$R_x = \int_0^{2a} dm (2ax \sin \alpha) \omega^2$$

$$= \int_0^{2a} \frac{m \sin \alpha \omega^2}{2a} x dx$$

$$R_x = m \omega^2 a \sin \alpha$$

$$mg \sin \alpha = \int dm \omega^2 x \sin \alpha x \cos \alpha$$

$$\Rightarrow \omega^2 = \frac{3g}{4a \cos \alpha}$$

$$R_x = \frac{3mg \tan \alpha}{4} = \frac{3R_y \tan \alpha}{4}$$

$$\frac{R_x}{R_y} = \tan \theta = \frac{3 \tan \alpha}{4}$$

$$\theta = \tan^{-1} \left(\frac{3}{4} \tan \alpha \right)$$

Q.1 (e) Find the equation of the streamlines for the flow $q = -(3y^2)\hat{i} - 6x\hat{j}$ at the point (1, 1). (10)

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{-3y^2} = \frac{dy}{-6x}$$

$$x^2 = \frac{y^3}{3} + C_1$$

at (1, 1)

$$C_1 = \frac{2}{3}$$

$$\Rightarrow \boxed{x^2 = \frac{y^3 + 2}{3}}$$

Q.2 (a) Find the general solution and singular solution of $6yz - 6pxy - 3qy^2 + pq = 0$. (20)

Non linear PDE.

Solution by Charpit's.

$$\frac{dx}{f_p} = \frac{dy}{f_q} = \frac{dp}{-f_x - pf_z} = \frac{dq}{-f_y - qf_z} = \frac{dz}{f}$$

$$\frac{dp}{6py - p(6y)} \Rightarrow dp = 0 \Rightarrow p = c.$$

$$\Rightarrow 6yz - 6cxy + q(c - 3y^2) = 0$$

$$\Rightarrow q = \frac{6(cxy - yz)}{c - 3y^2}$$

$$q = \frac{6y(cx - z)}{c - 3y^2}$$

$$dz = p dx + q dy$$

$$dz = c dx + \frac{6y}{c - 3y^2} (cx - z) dy$$

$$(c - 3y^2) dz + 6yz dy = c \left(dx + 6xy dy \right)$$

$$\frac{(c-3y^2)dy + 6xy^3 dy}{(c-3y^2)^2} = \frac{c((c-3y^2)dx + 6xydy)}{(c-3y^2)^2}$$

$$d\left(\frac{3}{c-3y^2}\right) = c d\left(\frac{x}{(c-3y^2)}\right)$$

$$\Rightarrow \frac{3}{c-3y^2} = \frac{cx}{c-3y^2} + d$$

$$\Rightarrow 3 = cx + d(c-3y^2)$$

$$\Rightarrow \boxed{3 = cx - d3y^2 + dc}$$

is general solution.

Check

$$p = c; \quad q = -6dy$$

$$\frac{dy}{dx} = -\frac{6(c-xy)}{c-3y^2}$$

$$6 \left[y(cx - 3dy^2 + dc) - xyc + 3dy^2 - cdy \right] = 0$$

Singular solution $\rightarrow$

$$\frac{\partial z}{\partial c} = 0 \Rightarrow x + d = 0$$

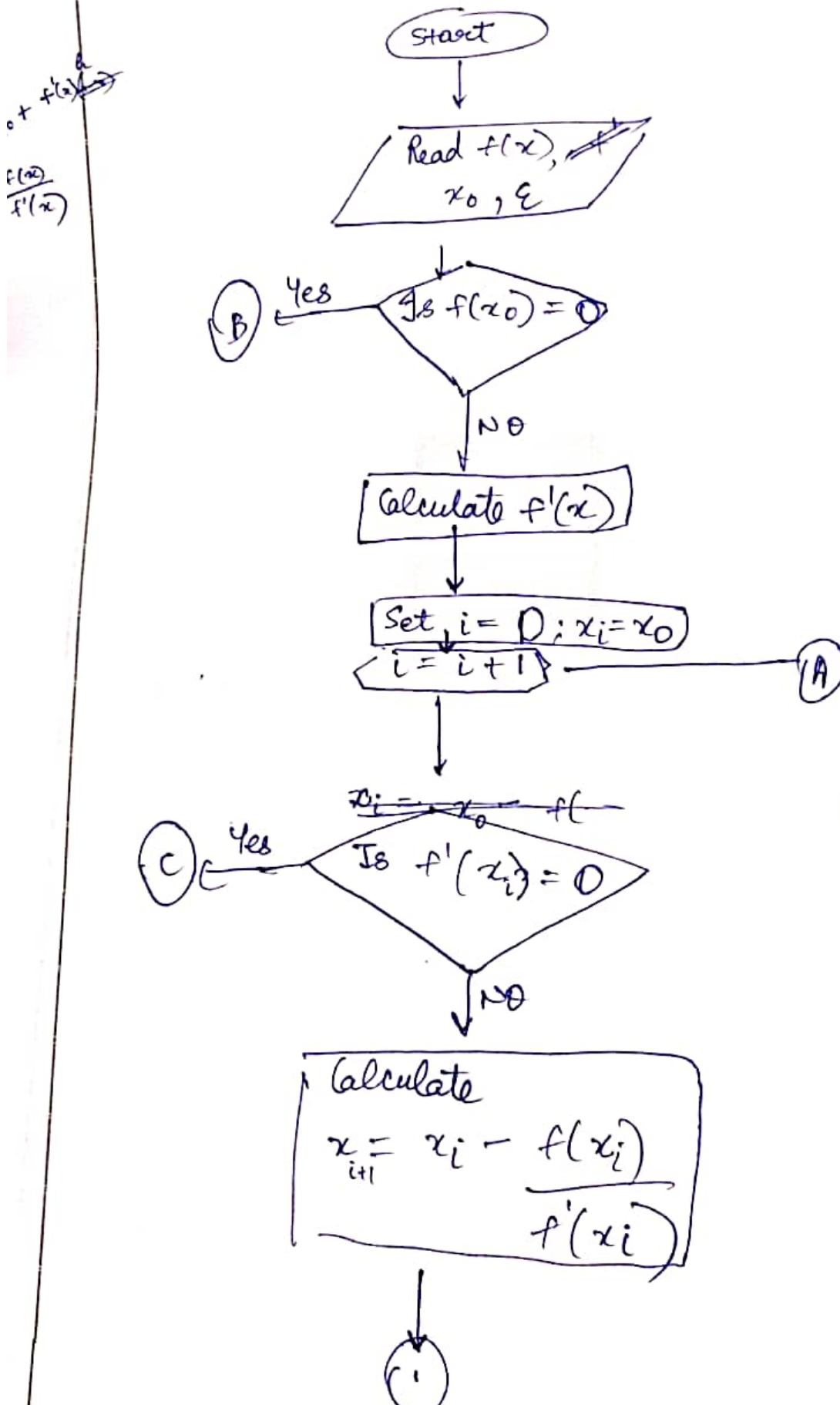
$$\Rightarrow d = -x$$

$$\frac{\partial z}{\partial d} = 0 \Rightarrow c - 3y^2 = 0 \Rightarrow c = 3y^2$$

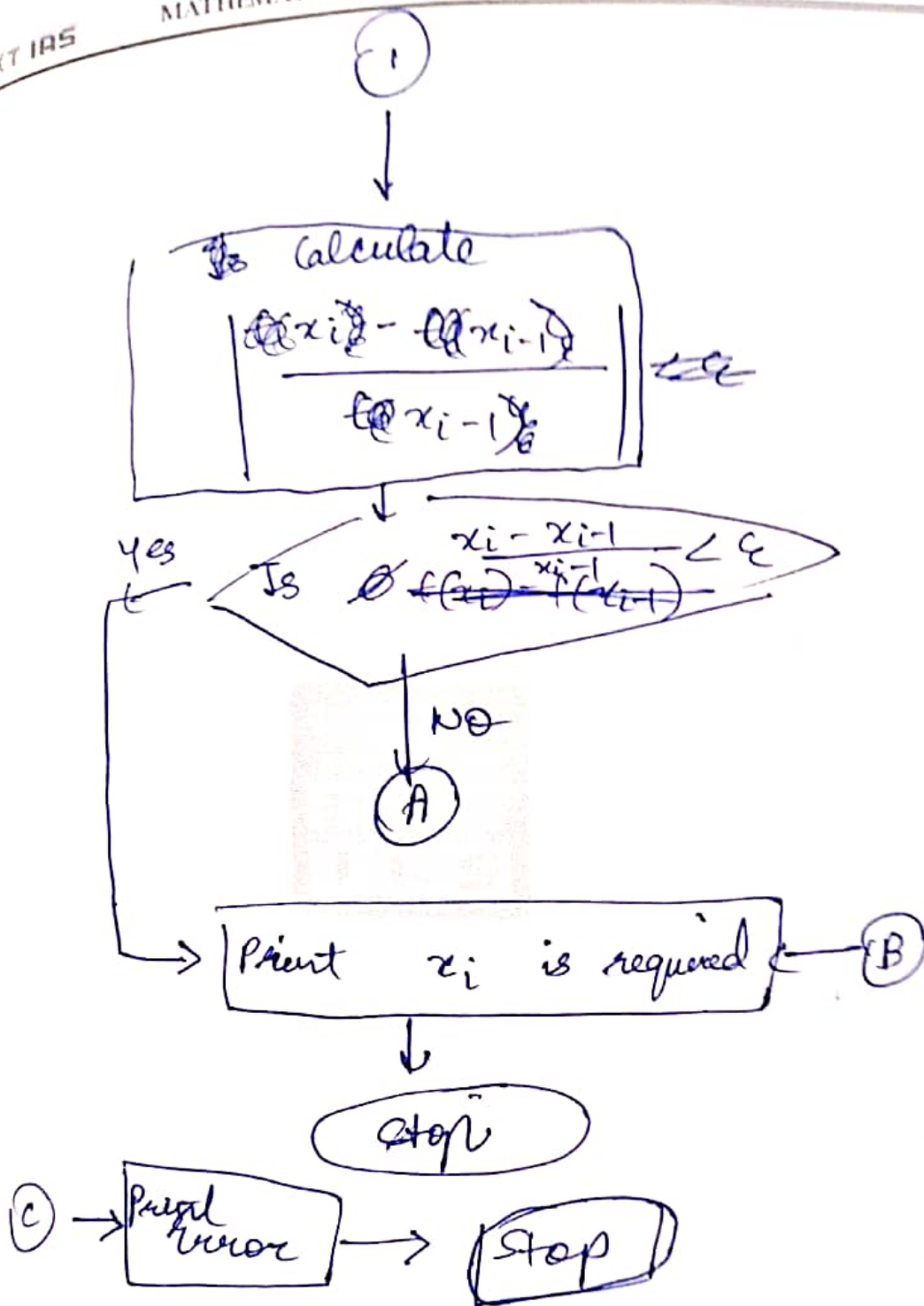
$$\Rightarrow z = 3y^2x + 3xy^2 - 3xy^2$$

$\Rightarrow \frac{1}{10-3y^2}x$ is singular

2. (b) Write an algorithm in the form of a flow chart for Newton-Raphson method. Describe the cases of failure of this method. (15)



Next IAS



2 (c) Show that the surface $\frac{x^2}{a^2 k^2 t^4} + k t^2 \left(\frac{y^2}{b^2} + \frac{z^2}{c^2} \right) = 1$ is a possible form of boundary surface of a liquid at time t . (15)

$$\phi: \frac{x^2}{a^2 k^2 t^4} + k t^2 \left(\frac{y^2}{b^2} + \frac{z^2}{c^2} \right) - 1$$

$$\frac{d\phi}{dt} = 0 = \frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} + w \frac{\partial \phi}{\partial z}$$

$$\frac{-4x^2}{a^2 k^2 t^5} + u \left[\frac{2x}{a^2 k^2 t^4} \right] + v \left[\frac{2y}{b^2} k t^2 \right] + w \left[\frac{2z}{c^2} k t^2 \right]$$

$$+ 2k t \left[\frac{y^2}{b^2} + \frac{z^2}{c^2} \right] = 0$$

$$\Rightarrow \frac{2x}{a^2 k^2 t^4} \left[u - \frac{2x}{t} \right] + \frac{2y}{b^2} k t^2 \left[v + \frac{y}{t} \right]$$

$$+ \frac{2z}{c^2} k t^2 \left[w + \frac{z}{t} \right] = 0$$

holds if $u = \frac{2x}{t}; v = -\frac{y}{t}; w = -\frac{z}{t}$

is possible if

$$\nabla \cdot \mathbf{q} = 0$$

i.e. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{2}{t} - \frac{1}{t} - \frac{1}{t} = 0$

Hence given surface is possible form of boundary.

Find a surface satisfying $r + s = 0$ and touching the elliptic paraboloid $z = 4x^2 + y^2$ along its section by the plane $y = 2x + 1$. (15)

$$r + s = 0.$$

$$(D^2 + DD')z = 0$$

$$D(D + D')z = 0$$

Auxiliary equation is

$$m(m+1) = 0$$

$$\Rightarrow m = 0, -1$$

$$\Rightarrow z = \phi_1(y) + \phi_2(y-x)$$

$$p = \phi_1'(y) + \phi_2'(y-x); q = -\phi_2'(y-x)$$

$$z = 4x^2 + y^2$$

$$p = 8x; q = 2y$$

$$\text{along } y = 2x + 1$$

$$\text{i.e. } (t, 2t+1)$$

$$p = p_1; q = q_1$$

$$\text{i.e. } \phi_1'(2t+1) = 8t; -\phi_2'(t+1) = 2(2t+1)$$

$$\Rightarrow \phi_1^*(2t+1) = 4t^2 + c_1; \phi_2(t+1) = -2(t^2+t) + c_2$$

$$\phi_1(t) = 4\left(\frac{t-1}{2}\right)^2 + c_1; \phi_2(t) = -2\left(\frac{t-1}{2}\right)^2 + (t-1) + c_2$$

$$\phi_1(t) = (t-1)^2 + c_1 \quad ; \quad \phi_2(t) = -2(t^2 - t + 2)$$

$$\phi_1(y) = (y-1)^2 + c_1 \quad ; \quad \phi_2(y-x) = -2[(y-x)^2 - (y-x) + 2] + c_2$$

$$\text{also } z = \phi_1(y) + \phi_2(y-x) = 4x^2 + y^2$$

$$\text{along } y = 2x + 1.$$

$$\text{i.e. along } (t, 2t+1)$$

$$\phi_1(2t+1) + \phi_2(t+1) = 4t^2 + (2t+1)^2$$

$$4t^2 + c_1 - 2(t^2 + t) + c_2 = 4t^2 + (2t+1)^2$$

$$\Rightarrow c_1 + c_2 = 6t^2 + 4t + 1 = 6x^2 + 4x + 1$$

$$\Rightarrow \phi_1(t) = (t-1)^2 + \phi_2(t) + 2(t^2 - t + 2)$$

$$\phi_1(y) + \phi_2(y-x) = (y-1)^2 + 2[(y-x)^2 - (y-x) + 2]$$

$$= c_1 + c_2 = 6x^2 + 4x + 1$$

$$\Rightarrow \phi_1(y) + \phi_2(y-x) = (y-1)^2 + 2[y^2 - y - 2xy]$$

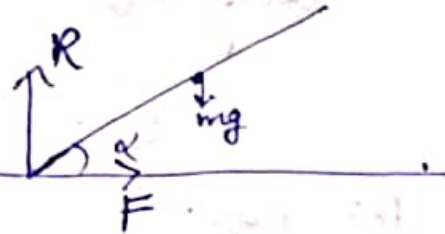
$$4x^2 + 2x - 3$$

$$\Rightarrow z = -y^2 - 2 + 4xy + 2x + 4x^2.$$

Q.4 (b)

A uniform rod is placed with one end in contact with a horizontal table and is then inclined at an angle of α to the horizon and is allowed to fall. When it becomes horizontal, show that the angular velocity is $\sqrt{\frac{3g}{2a}} \sin \alpha$ whether the plane be perfectly smooth or perfectly rough. Show that the end of the rod will not leave the plane in either case. (20)

Case I - perfectly rough
let F is friction
 $F = \mu R$



$R = mg$ (over)

$$m \frac{d^2 x}{dt^2} = F = \mu R$$

$$m \frac{d^2 y}{dt^2} = R - mg$$

$$y = \frac{a}{2} \sin \theta$$

$$m \frac{d^2}{dt^2} \left(\frac{a}{2} \sin \theta \right) = \frac{ma}{2} \frac{d}{dt} (\cos \theta \dot{\theta})$$

$$= \frac{ma}{2} (-\sin \theta \dot{\theta}^2 + \cos \theta \ddot{\theta})$$

$$\frac{1}{2} M (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} M k^2 \dot{\theta}^2 = W g$$

$$x = \frac{a}{2} \cos \theta$$

$$\frac{1}{2} M \left(\frac{a^2}{4} \dot{\theta}^2 + \frac{a^2}{12} \dot{\theta}^2 \right) = M g \frac{a}{2} (\sin \alpha - \sin \theta)$$

$$\Rightarrow \dot{\theta}^2 = \frac{3g}{a} (\sin \alpha - \sin \theta) \quad \text{--- ii}$$

When $\theta = 0$

$$\dot{\theta} = \sqrt{\frac{3g}{a} \sin \alpha}$$

Using $\dot{\theta}$ in From i and ii

$$2\dot{\theta}\ddot{\theta} = -\frac{3g}{a} \cos \theta \dot{\theta}$$

$$R = Mg$$

$$\Rightarrow \ddot{\theta} = -\frac{3g}{2a} \cos \theta \quad \text{--- iii}$$

from i, ii, iii

$$R = \frac{mg}{4} (6 \sin^2 \theta - 3 \cos^2 \theta + 4 - 6 \sin \alpha \sin \theta)$$

$$R = \frac{mg}{4} f(\theta)$$

$$f'(\theta) = 12 \sin \theta \cos \theta + 6 \cos \theta \sin \alpha - 6 \sin \alpha \cos \theta$$

$$f'(\theta) = 6 \cos \theta [3 \sin \theta + \sin \alpha - \sin \alpha]$$

$$\cos \theta > 0$$

$$f'(\theta) = 0 \Rightarrow \sin \theta = \frac{\sin \alpha}{3}$$

$$\cos^2 \theta = \frac{9 - \sin^2 \alpha}{9}$$

$$f(\theta) = \sin^2 \alpha \left[\frac{6}{9} \right] - 3 \left[1 - \frac{\sin^2 \alpha}{9} \right] + 4 - 6 \frac{\sin^2 \alpha}{3}$$

$$1 - \sin^2 \alpha = \cos^2 \alpha$$

$$\cancel{f(\theta)}$$

$$f''(\theta) = 18 \cos 2\theta + 6 \sin \alpha \cos \theta$$

$$f''(\theta) > 0$$

$$\Rightarrow f(\theta) = \cos^2 \alpha \text{ is minimum}$$

$$f(\theta) > \cos^2 \alpha > 0$$

$$\Rightarrow R > 0$$

Case II Perfectly smooth.

$$\frac{1}{2} M (\dot{y}^2) + \frac{1}{2} M R^2 \dot{\theta}^2 = Wq$$

$$\dot{\theta}^2 \left(\cos^2 \theta + \frac{1}{3} \right) = \frac{4g}{a} (\sin \alpha - \sin \theta)$$

$$\theta = 0 \Rightarrow \dot{\theta} = \sqrt{\frac{3g}{a} \sin \alpha}$$

$$2\ddot{\theta}(\cos^2\theta + \frac{1}{3}) - 2\cos\theta\sin\theta\dot{\theta}(\dot{\theta}^2) = -\frac{4g\cos\theta}{a}\dot{\theta}$$

$$\ddot{\theta} = \frac{2\sin 2\theta \dot{\theta}^2 - \frac{4g\cos\theta}{a}}{2(\cos^2\theta + \frac{1}{3})}$$

$$\dot{\theta} = \frac{\sin 2\theta \left[\frac{4g(\sin\theta - \sin\theta)}{a(\cos^2\theta + \frac{1}{3})} \right] - \frac{4g\cos\theta}{a}}{2(\cos^2\theta + \frac{1}{3})}$$

$$\tau = I\alpha$$

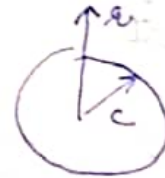
$$R \frac{a \cos\theta}{2} = \frac{ma^2}{12} \ddot{\theta}$$

$$R = \frac{ma^2 \ddot{\theta}}{6 \cos\theta}$$

$$R = \frac{ma}{6 \cos\theta} \begin{bmatrix} \ddot{\theta} \\ \dot{\theta} \end{bmatrix} > 0$$

An infinite mass of fluid is acted on by a force $\frac{\mu}{r^{3/2}}$ per unit mass directed to the origin. If initially, the fluid is at rest and there is cavity in the form of the sphere $r = c$ in it, show that the cavity will be filled up after an interval of time $\left(\frac{2}{5\mu}\right)^{1/2} c^{5/4}$. (15)

$$r^2 u = f(t) \\ \Rightarrow u' = \frac{f'(t)}{r^2}$$



$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = F - \frac{1}{\rho} \frac{\partial p}{\partial r}$$

$$\frac{f'(t)}{r^2} + \frac{1}{2} \left(\frac{\partial u^2}{\partial r} \right) = -\frac{\mu}{r^{3/2}} - \frac{\partial}{\partial r} \left(\frac{p}{\rho} \right)$$

Integrating w.r.t. r

$$-\frac{f'(t)}{r} + \frac{u^2}{2} = \frac{2\mu}{r^{1/2}} - \frac{p}{\rho} + C_1$$

$$\text{at } r \rightarrow \infty; u = 0; p = 0$$

$$\Rightarrow C_1 = 0$$

$$f(t) = r^2 u$$

let r_1 be radius of cavity at time t
at $r = r_1$; $p = 0$;
 $u = u_1$

$$-\frac{f'(t)}{r_1} + \frac{u_1^2}{2} = \frac{2\mu}{r_1^{1/2}}$$

$$\frac{df}{dt} = 2r \left(\frac{dr}{dt} \right) u + r^2 \frac{du}{dt} \quad \text{at } r = r_1, u = u_1 \\ = 2r_1 u_1^2 + r_1^2 \frac{du_1}{dt}$$

$$-2u_1^2 - r_1 \frac{du_1}{dt} + \frac{u_1^2}{2} = \frac{2\mu}{r_1^{1/2}}$$

$$-r_1 \frac{du_1}{dt} - \frac{3}{2} u_1^2 = \frac{2\mu}{r_1^{1/2}}$$

$$-r_1 \frac{d^2 r_1}{dt^2} - \frac{3}{2} \left(\frac{dr_1}{dt} \right)^2 = \frac{2\mu}{r_1^{1/2}}$$

$$-r_1 \frac{2}{dt} \left(\frac{dr_1}{dt} \right) \frac{dr_1}{dt^2}$$

$$-\frac{3u_1^2}{2} - r_1 u_1 \frac{du_1}{dr} = \frac{2\mu}{r_1^{1/2}}$$

$$r_1 u_1 du_1 = - \left[\frac{3u_1^2}{2} + \frac{2\mu}{r_1^{1/2}} \right] dr$$

$$\frac{3}{2} \int 2 r_1^{3/2} u_1 du_1 + 3 u_1^2 r_1^{1/2} dr + 4 \mu dr = 0$$

$$d(u_1^2 r_1^3) + 4 \mu r_1^{3/2} dr = 0$$

$$u_1^2 r_1^3 + \frac{8\mu}{5} r_1^{5/2} + C_1 = 0$$

$$\text{at } r_1 = C; v = 0 \Rightarrow$$

$$r_1^3 v_1^2 = \frac{8\mu}{5} \left(C^{5/2} - r_1^{5/2} \right)$$

$$\frac{dr}{dt} = v = \left(\frac{8\mu}{5r_1^3} \right)^{1/2} \sqrt{c^{5/2} - r_1^{5/2}}$$

$$dt = \frac{dr (5r_1^{3/2})^{1/2}}{(8\mu)^{1/2} \sqrt{c^{5/2} - r_1^{5/2}}}$$

$$c^{5/2} - r_1^{5/2} = x^2$$

$$\int_0^t dt = \int_0^c \frac{-4k d\alpha}{(8\mu)^{1/2} \alpha^{5/2}} \quad \text{where } \alpha = r_1^{5/2}$$

$$t = \frac{4}{(8\mu)^{1/2}} \left(-\frac{1}{2\sqrt{2} \alpha^{1/2}} \right) \Big|_0^c = \frac{4}{(8\mu)^{1/2}} \left(-\frac{1}{2\sqrt{2} c^{1/2}} \right) = \sqrt{2} c$$

$$t = \left(\frac{2}{5\mu} \right)^{1/2} c^{5/4}$$

SECTION-B

- Q.5 (a) Transform the equation $y^2z_x - xz_y = 0$ into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of revolution. (10)

$$y^2z_x - xz_y = 0$$

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$r^2 = x^2 + y^2 + z^2$$

$$\tan \theta = \frac{\sqrt{x^2 + y^2}}{z}$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial z}{\partial \theta} \frac{\partial \theta}{\partial x}$$

$$= (\cos \theta) \cdot \frac{x}{r} + (-r \sin \theta) \frac{\frac{2x}{r^2} \sec^2 \theta}{\sqrt{\frac{x^2 + y^2}{z^2}}}$$

$$= \cos \theta \sin \theta \cos \phi + \frac{(-r \sin \theta) \left(\frac{2x \sin \theta \cos \phi}{r^2 \cos^2 \theta} \right) \sec^2 \theta}{z + \tan \theta}$$

$$z_x = \cos \theta \sin \theta \cos \phi - \frac{\sin^2 \theta \cos \phi}{\cos^3 \theta \tan \theta}$$

$$z_y = \frac{\partial z}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial z}{\partial \theta} \frac{\partial \theta}{\partial y}$$

$$= (\cos \theta) \frac{y}{r} + (-r \sin \theta) \frac{\frac{2y}{r^2} \sec^2 \theta}{z + \tan \theta}$$

$$= \cos \theta \sin \theta \sin \phi - \frac{r}{\cos \theta} \left(\frac{r \sin \theta \sin \phi}{r^2 \cos^2 \theta} \right)$$

$$y z_x - x z_y = r \sin \theta (\sin \phi z_x - \cos \phi z_y)$$

$$= r \sin \theta \left(- \frac{\sin \phi \cos \phi \sin \theta}{\cos^3 \theta} + \frac{\sin \theta \sin \phi}{\cos^3 \theta} \right)$$

$$= 0$$

$$\Rightarrow y z_x = x z_y$$

$$x = r \cos \theta ; \quad y = r \sin \theta$$

$$z_x = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial z}{\partial \theta} \frac{\partial \theta}{\partial x}$$

$$= z_r \frac{r}{x} + \frac{\partial z}{\partial \theta} \left(\frac{1}{r \cos \theta} \left(\frac{-r \sin \theta}{r^2 \cos^2 \theta} \right) \right)$$

$$= z_r \cos \theta - \frac{z_\theta \sin \theta}{r}$$

$$z_y = z_r \sin \theta - \frac{z_\theta \cos \theta}{r}$$

$$y z_x - x z_y = z_r (r \sin \theta \cos \theta) - \left(z_\theta \sin^2 \theta \right) - z_r (r \sin \theta \cos \theta) + z_\theta \cos^2 \theta = 0$$

$$\Rightarrow z_\theta \cos 2\theta = 0 \Rightarrow z_\theta = 0 \Rightarrow z \text{ is revolution surface}$$

Q.5 (b)

Verify that the equation $yz(y+z)dx + xz(x+z)dy + xy(x+y)dz = 0$ is integrable and find its solution. (10)

$$\frac{dx}{yz(y+z)} = \frac{dy}{xz(x+z)} = \frac{dz}{xy(x+y)}$$

$$= \frac{xdx + ydy + zdz}{xyz}$$

$$Mdx + Ndy + Ldz = 0$$

$$\nabla \times (M\hat{i} + N\hat{j} + L\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz(y+z) & xz(x+z) & xy(x+y) \end{vmatrix}$$

$$\hat{i} (x^2 + 2xy - x^2 - 2xz)$$

$$yz(y+z)$$

- 5 (c) Express the Boolean function $F(x, y, z) = xy + x'z$ in a product of maxterm form. (10)

Max term = Product of sum
 $xy + x'z$

$$= (xy + x')(xy + z)$$

$$(a+bc = (a+b)(a+c))$$

$$a = xy$$

$$b = x'$$

$$c = z$$

$$= [(x+x')(y+x')][(z+x)(z+y)]$$

$$= (y+x')(z+x)(z+y)$$

$$= \overline{P} \overline{Q} \overline{R}$$

$$= (x' + y + zz')(x + z + yy')(xx' + y + z)$$

$$= (x' + y + z)(x' + y + z')(x + z + y)(x + z + y')$$

$$(x + y + z)(x' + y + z)$$

$$= (x' + y + z)(x + y + z)(x' + y + z)$$

$$= \Pi M(0, 5).$$

Q.5 (d) Convert $(3798.3875)_{10}$ into octal and hexadecimal equivalents

(114)

$$(3798.3875)_{10}$$

∴ Octal .

8	3798	~~474~~ 6
	4748	2
	59	3
	7	7
	0	

↑

$$(3798)_{10} = (7326)_8$$

$$0.3875 \times 8 = 3.1$$

$$0.1 \times 8 = 0.8$$

$$0.8 \times 8 = 6.4$$

$$0.4 \times 8 = 3.2$$

$$0.2 \times 8 = 1.6$$

$$0.6 \times 8 = 4.8$$

$$0.8 \times 8 = 6.4$$

3
0
6
3
1
4
6

↓

$$(0.3875)_{10} = (0.306314)_8$$

$$(3798.3875)_{10} = (7326.306314)_8$$

ii) Hexadecimal .

$$\begin{array}{r}
 16 \overline{) 3798} \\
 \underline{237} \\
 14 \\
 0
 \end{array}
 \begin{array}{l}
 6 \\
 13 = D \\
 14 = E
 \end{array}
 \begin{array}{c}
 \uparrow
 \end{array}$$

(ED6)

$$0.3875 \times 16 = 6.2$$

$$0.2 \times 16 = 3.2$$

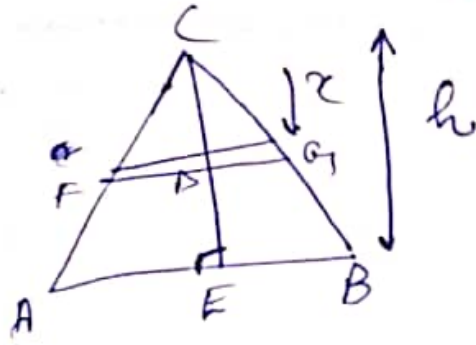
$$0.2 \times 16 = 3.2$$

$\begin{array}{c} 6 \\ 3 \\ 3 \end{array}$
 $\downarrow$

Ans (ED6 - 63)₁₆ .

Find the moment of inertia of triangular lamina.

(10)



$$DE = h - x$$

$$I_{AB} = I_D + m(h-x)^2$$

$$\frac{FG}{AB} = \frac{x}{h} \Rightarrow FG = \frac{ax}{h}$$

$$m_{FG} = \frac{m}{\frac{1}{2} \times a \times h} \left(\frac{ax}{h} \right) dx$$

$$I_{AB} = \int_0^h \frac{m}{\frac{1}{2} \times a \times h} \left(\frac{ax}{h} \right) dx (h-x)^2$$

$$= \frac{2m}{ah^2} \int_0^h x(h-x)^2 dx$$

$$= \frac{2m}{ah^2} \left[\frac{h^4}{2} + \frac{h^4}{4} - \frac{2}{3} h^3 \right]$$

$$= 2mh^2 \left[\frac{3}{4} - \frac{2}{3} \right] = \left[\frac{mh^2}{6} \right]$$

(15)

Q.6 (a) Solve: $r+s-2t=(2x+y)^{1/2}$.

$$r+s-2t=(2x+y)^{1/2}$$

$$(D^2 + DD' - 2D'^2)z = (2x+y)^{1/2}$$

$$(D+2D')(D-D')z = (2x+y)^{1/2}$$

Auxiliary equation is

$$(m+2)(m-1)=0$$

$$m = -2, 1$$

$$z_c = \phi_1(y-2x) + \phi_2(y+x)$$

$$z_p = \frac{1}{(D+2D')(D-D')} (2x+y)^{1/2}$$

$$(D+2D')(D-D')$$

$$F = \frac{1}{(D+2D')(D-D')} (1-D')^{-1} (2x+y)^{1/2}$$

$$z_p = \frac{1}{(D+2D')(2-1)} \int (2x+y)^{1/2} du$$

$$z_p = \frac{2}{3} \frac{1}{D+2D'} (2x+y)^{3/2}$$

$$z_p = \frac{2}{3} (2x+y)^{3/2}$$

$$z_p = \frac{2}{3} \frac{(2x+y)^{5/2}}{5/2 (2+2)} = \frac{5}{3} (2x+y)^{5/2}$$

$$z = z_p + z_c$$

$$= \phi_1(y-2x) + \phi_2(y+x) + \frac{5}{3} (2x+y)^{5/2}$$



Use Gauss Quadrature formula of six points to evaluate $\int_0^1 \frac{dx}{1+x^2}$, given

$$x_1 = -0.23861919, w_1 = 0.46791393$$

$$x_2 = -0.66120939, w_2 = 0.36076157$$

$$x_3 = -0.93246951, w_3 = 0.17132449$$

(15)

$$x_4 = -x_1, x_5 = -x_2, x_6 = -x_3, w_4 = w_1, w_5 = w_2, \text{ and } w_6 = w_3.$$

$$I = \int_0^1 \frac{dx}{1+x^2}$$

$$\text{where } f(x) = \frac{1}{1+x^2}$$

$$I = \sum_{i=1}^6 w_i f(x_i)$$

$$= 0.46791393 \left(\frac{1}{1+0.23861919^2} \right)$$

$$= 2 \sum_{i=1}^3 w_i f(x_i)$$

$$\text{as } f(-x_i) = f(x_i)$$

$$\text{let } u = ax + b$$

$$\text{such that } x=0 \Rightarrow u=-1$$

$$x=1 \Rightarrow u=1$$

$$-1 = b; \quad 1 = a + b \Rightarrow a = 2$$

$$u = 2x - 1 \Rightarrow dx = \frac{du}{2}$$

$$I = \frac{1}{2} \int_{-1}^1 \frac{du}{1 + \left(\frac{u+1}{2}\right)^2}$$

$$= \frac{1}{2} \sum_{i=1}^6 w_i f(x_i) \quad \text{where } x_i = x_i$$

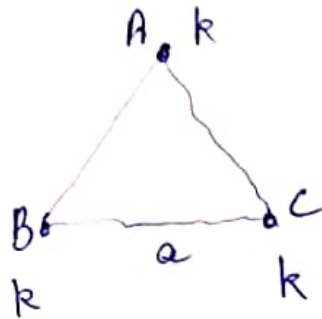
$$\text{where } f(u) = \frac{1}{1 + \left(\frac{u+1}{2}\right)^2}$$

$$I = \frac{1}{2} (1.57079629)$$

$$= 0.7853981449$$

6 (c)

Three parallel rectilinear vortices of the same strength k and in the same sense meet any plane perpendicular to them in an equilateral triangle of side a . Show that all the vortices move round the same cylinder with uniform speed in time $(4\pi^2 a^2) / 3k$. (20)



$$w_A = \frac{ik}{2\pi} \log(z - z_B) + \frac{ik}{2\pi} \log(z - z_C)$$

$$\frac{dw_A}{dz} = \frac{ik}{2\pi} \left[\frac{1}{z - z_B} + \frac{1}{z - z_C} \right]$$

$$\left. \frac{dw_A}{dz} \right|_{z=z_A} = \frac{ik}{2\pi} \left[\frac{2z_A - z_B - z_C}{(z_A - z_B)(z_A - z_C)} \right]$$

$$\text{let } z_B = z_B$$

$$z_A = z_B + ae^{i\pi/3}$$

$$z_C = z_B + a$$

$$\left. \frac{dw_A}{dz} \right|_{z=z_A} = \frac{ik}{2\pi} \left[\frac{2ae^{i\pi/3} - a}{ae^{i\pi/3}(ae^{i\pi/3} - a)} \right]$$

$$= \frac{ik}{2\pi} \left[\frac{(i\sqrt{3}a) e^{-i\pi/3}}{a \left(-\frac{a}{2} + i\frac{a\sqrt{3}}{2} \right)} \right]$$

$$\left| \frac{dw_A}{dz} \right|_{z=z_A} = \frac{k}{2\pi} \frac{\sqrt{3}a}{a \sqrt{\frac{a^2}{4} + \frac{\sqrt{3}a^2}{4}}}$$

$$\left| \frac{dw_A}{dz} \right|_{z=z_A} = \frac{\sqrt{3}k}{2\pi a}$$

Also

$$\begin{aligned} \frac{dw_A}{dz} &= -\frac{\sqrt{3}}{2\pi a} \left[\frac{e^{-i\pi/3}}{e^{i(\pi-\pi/3)}} \right] \\ &= -\frac{\sqrt{3}}{2\pi a} \left[e^{-i\pi} \right] = \frac{\sqrt{3}}{2\pi a} \end{aligned}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{2\pi a/\sqrt{3}}{\sqrt{3}k/2\pi a} = \frac{4\pi^2 a^2}{3k}$$