Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

NEXT IAS

(To be filled by candidate)

Name of Candidate : VAIBHAV PANDEY

Roll No. : OAC24MTTN004

Registration Number : NIAS2400020189 Date of Examination : 7/7/24

Exam Centre : Old Rajinder Nagar ☐ Online ☑

CSE (MAINS) TEST SERIES - 2024

Test-01	MATHEMATICS OPTIONAL
Dated : 23-06-2024	

Time Allowed : Three Hours

Maximum Marks: 250

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are ***EIGHT*** *questions divided in* ***Two sections*** *and printed in* ***ENGLISH.***

Candidate has to attempt ***FIVE*** *questions in all.*

Question No. 1 and 5 *are compulsory and out of the remaining, any* ***THREE*** *are to be attempted choosing at least* ***ONE*** *question from each section.*

The number of marks carried by a question/part is indicated against it.

Answers must be written in the medium authorized in the Admission. Certificate which must be stated clearly on the cover of this Question-cum- Answer (QCA) booklet in the space provided.

No marks will be given for answers written in a medium other than the authorized one.

Word limit in questions, wherever specified, should be adhered to.

Attempts of question shall be counted in sequential order, unless struck off. Attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question cum Answer (QCA) booklet must be clearly struck off.

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

(For filling by Examiners only)

Q. No.	Page No.	Max. Marks	Marks	Total	Signature
1. (a)	4	10			
1. (b)	6	10			
1. (c)	8	10			
1. (d)	10	10			
1. (e)	12	10			
2. (a)	14	20			
2. (b)	18	15			
2. (c)	21	15			
3. (a)	24	15			
3. (b)	27	15			
3. (c)	30	20			
4. (a)	34	15			
4. (b)	37	15			
4. (c)	40	20			
5. (a)	44	10			
5. (b)	46	10			
5. (c)	48	10			
5. (d)	50	10			
5. (e)	52	10			
6. (a)	54	15			
6. (b)	57	20			
6. (c)	61	15			
7. (a)	64	15			
7. (b)	67	15			
7. (c)	70	20			
8. (a)	74	15			
8. (b)	77	20			
8. (c)	81	15			
Grand Total					

Remarks

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Observations:

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

SECTION-A

Q.1 (a) If V is finite dimensional and f is a homomorphism of V onto V, prove that f must be one one and so, an isomorphism. **(10)**

Let $\dim V = n$ and $\alpha (\neq \bar{0}) \in V$.

Let $f(\alpha) = \bar{0}$

$\because \dim V = n$, we can form a basis of V by extending the set $\{\alpha\}$.

Let $S = \{\alpha, \alpha_1, \alpha_2, \cdots, \alpha_{n-1}\}$ is a basis of V

For any $\beta \in V$, we can write

$$\beta = a\alpha + a_1\alpha_1 + a_2\alpha_2 + \cdots a_{n-1}\alpha_{n-1}$$

where $a, a_1, \cdots a_{n-1} \in F$

(F is the field over which Vector space V is defined)

$$\therefore f(\beta) = f(a\alpha + a_1\alpha_1 + \cdots a_{n-1}\alpha_{n-1})$$

$$= a f(\alpha) + a_1 f(\alpha_1) + \cdots + a_{n-1} f(\alpha_{n-1})$$

= linear combination of

$f(\alpha_1), f(\alpha_2), \cdots f(\alpha_{n-1})$. $\{\because f(\alpha) = \bar{0}\}$

$\therefore$ The range of f is spanned by

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

$\{ f(\alpha_1), f(\alpha_2) \text{ ---- } f(\alpha_{n-1}) \}$.

$\therefore$ dim (Range f) $\leq n-1$

$\therefore$ Range f $\neq V$ $\{\because \dim V = n\}$

$\therefore$ f is <u>not onto</u>.

$\therefore$ our assumption that $f(\alpha) = \overline{0}$ for $\alpha \neq \overline{0}$ is wrong

$\Rightarrow f(\alpha) = \overline{0} \Rightarrow \alpha = \overline{0}$ — (1)

Now, let $f(\alpha) = f(\beta)$ $\{\alpha, \beta \in V\}$

$\Rightarrow f(\alpha) + (-f(\beta)) = \overline{0}$

$\Rightarrow f(\alpha + (-\beta)) = \overline{0}$ $\{\because f(-\beta) = -f(\beta)$ for homomorphism$\}$

$\Rightarrow \alpha - \beta = \overline{0}$ {from (1)}

$\Rightarrow \alpha = \beta$

$\therefore$ f is <u>one-one</u>.

$\therefore$ f is both one-one and onto.

$\therefore$ f is an **isomorphism**.

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Q.1 (b) Determine the set of points where the function $f(x)=\frac{x}{1+|x|}$ is differentiable. **(10)**

$$f(x) = \frac{x}{1+x} \quad \text{for } x \geq 0$$

$$= \frac{-x}{1-x} \quad \text{for } x < 0$$

Clearly, $f(x)$ is differentiable $\forall x \neq 0$

for $x = 0$,

$$R.H.D. = \lim_{h \to 0} \frac{\frac{h}{1+h} - 0}{h}$$

$$= 1$$

$$L.H.D. = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{\frac{-(-h)}{1-(-h)} - 0}{-h}$$

$$= \lim_{h \to 0} \frac{1}{-(1+h)}$$

$$= -1$$

$\therefore$ L.H.D $\neq$ R.H.D. $\Rightarrow$ f is not differentiable at $x = 0$.

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

$\therefore$ $f(x)$ is differentiable $\forall x \neq 0$

and not differentiable for $x = 0$.

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Q.1 (c) Evaluate $\lim_{x\to 0}\left(\frac{1}{x^2}-\cot^2 x\right)$ (10)

$$= \lim_{x\to 0}\left(\frac{1}{x^2}-\frac{\cos^2 x}{\sin^2 x}\right)$$

$$= \lim_{x\to 0}\frac{\sin^2 x - x^2\cos^2 x}{x^2\sin^2 x}\cdot\frac{x^2}{x^2}\quad\left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 0}\frac{2\sin x\cos x - \left\{x^2(2\cos x)(-\sin x) + 2x\cos^2 x\right\}}{4x^3}$$

{ L' Hospital rule & $\lim_{x\to 0}\frac{x^2}{\sin^2 x}=1$ }

$$= \lim_{x\to 0}\frac{\sin x\cos x + x^2\sin x\cos x - x\cos^2 x}{2x^3}$$

$$= \lim_{x\to 0}\frac{(\sin x\cos x - x\cos^2 x)}{2x^3} + \frac{1}{2}$$

$$= \lim_{x\to 0}\frac{-\sin^2 x + \cos^2 x - \cos^2 x + x(2\cos x)\sin x}{6x^2}$$

$$+\frac{1}{2}$$

{ L' Hospital rule }

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

$$= \frac{-1}{6} + \frac{2}{6} + \frac{1}{2}$$

$$= \frac{2}{3}$$

$$\therefore \boxed{\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x \right) = \frac{2}{3}}$$

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Q.1 (d) Find the centre of the conic $\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{4}=1, 2x+2y-z=3$. **(10)**

Let (α, β, γ) be the centre of the conic.

Locus of chords of $\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{4}=1$ that are bisected at (α, β, γ) is given by $T = S_1$,

that is $\left(\frac{\alpha x}{9}+\frac{\beta y}{16}+\frac{\gamma z}{4}-1\right)$

$= \left(\frac{\alpha^2}{9}+\frac{\beta^2}{16}+\frac{\gamma^2}{4}-1\right)$

which is the plane of the conic

$\Rightarrow \frac{\alpha x}{9}+\frac{\beta y}{16}+\frac{\gamma z}{4} = \frac{\alpha^2}{9}+\frac{\beta^2}{16}+\frac{\gamma^2}{4}$.

which is same as $2x+2y-z=3$

$\Rightarrow \frac{\frac{\alpha}{9}}{2} = \frac{\frac{\beta}{16}}{2} = \frac{\frac{\gamma}{4}}{-1}$

$= \frac{\frac{\alpha^2}{9}+\frac{\beta^2}{16}+\frac{\gamma^2}{4}}{3}$

$\Rightarrow \beta = \frac{16\alpha}{9}, \quad \gamma = \frac{-2\alpha}{9}$

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

$$\therefore \frac{\alpha}{18} = \frac{\frac{\alpha^2}{9} + \frac{16\alpha^2}{81} + \frac{\alpha^2}{81}}{3}$$

$$\Rightarrow \frac{\alpha}{6} = \frac{26\alpha^2}{81}$$

$$\Rightarrow \alpha = 0 \quad \text{or} \quad \alpha = \frac{27}{52}$$

For $\alpha = 0$, we get $\beta = 0$, $\gamma = 0$

But $(0,0,0)$ does not lie on

$$2x + 2y - z = 3$$

$\therefore$ rejected.

For $\alpha = \frac{27}{52}$, $\beta = \frac{16\alpha}{9} = \frac{12}{13}$

$$\gamma = \frac{-2\alpha}{9} = \frac{-3}{26}.$$

$\therefore$ Centre of the conic is

$$\boxed{(\alpha, \beta, \gamma) \equiv \left(\frac{27}{52}, \frac{12}{13}, \frac{-3}{26}\right)}$$

Next IAS ID: QAC24MTTN004, MATHS001, 08-07-2024 04:04 AM

Candidates must not write on this margin

Q.1 (e) Find the equation of the cylinder whose generators touch the sphere $x^2+y^2+z^2=9$ and are perpendicular to the plane $x-y-3z=5$. (10)

We need to find enveloping cylinder to the given sphere that has generators with d.r's $(1,-1,-3)$.

$\therefore$ required cylinder is given by

$$SS_1 = t^2$$

$$\Rightarrow (x^2+y^2+z^2-9)\left(1^2+(-1)^2+(-3)^2\right) = (1\cdot x + (-1)\cdot y + (-3)\cdot z)^2$$

$$\Rightarrow 11(x^2+y^2+z^2-9) = (x-y-3z)^2$$

$$\Rightarrow 10x^2+10y^2+2z^2+2xy-6yz+6zx-99=0.$$

is the required cylinder.

Candidates must not write on this margin

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Q.2 (a) Let V be vector space of all functions from R into R; let V_e be the set of all even functions, $f(-x)=f(x)$; let V_o be the subset of odd functions, $f(-x)=-f(x)$.

i. Prove that V_e and V_o are subspaces of V.

ii. Prove that $V_e + V_o = V$

iii. Prove that $V_e \cap V_o = \{0\}$. (20)

(i) Consider the set V_e

$f(x)$ s.t. $f(x) = 0 \quad \forall x \in V_e$

$\therefore V_e$ is non-empty.

Let $g(x), h(x) \in V_e$

Let $p(x) = ag(x) + bh(x)$ where $a, b \in R$

Now, $p(-x) = ag(-x) + bh(-x)$

$= ag(x) + bh(x) \quad \{\because g, h \in V_e\}$

$= p(x)$

$\Rightarrow p(x) \in V_e$

$\therefore V_e$ is subspace of V.

Consider the set V_o

$f(x)$ s.t. $f(x) = 0 \quad \forall x \in V_o$

$\therefore V_o$ is non empty

Let $g_1(x), h_1(x) \in V_o$

Next IAS ID: QAC24MTTN004_MATHS001_08-07-2024 04:01 AM
Candidates must not write on this margin

Let $q(x) = a g_1(x) + b h_1(x)$ where $a, b \in \mathbb{R}$

$\therefore q(-x) = a g_1(-x) + b h_1(-x)$

$= -a g_1(x) + (-b h_1(x))$

$\{\because g_1, h_1 \in V_0\}$

$= -(a g_1(x) + b h_1(x))$

$= -q(x)$

$\therefore q(x) \in V_0$

$\therefore V_0$ is subspace of V.

(ii) $\because V_e \subseteq V$ and $V_0 \subseteq V$

Also, V is a vector space satisfying closure property

$\therefore$ for $\alpha \in V_e$, $\beta \in V_0$

$\Rightarrow \alpha, \beta \in V$

$\therefore \alpha + \beta \in V$ (closure in V)

$\therefore V_e + V_0 \subseteq V$ — (1)

Now, let $f(x) \in V$

$\therefore f(x) = \underbrace{\frac{(f(x) + f(-x))}{2}}_{g(x)} + \underbrace{\frac{(f(x) - f(-x))}{2}}_{h(x)}$

Next IAS ID: OAC24MTTN004_MATHS001_08-07-2024 04:01 AM
Candidates must not write on this margin

$$g(-x) = \frac{f(-x) + f(x)}{2} = g(x) \quad \forall x \in \mathbb{R}$$

$\therefore g(x) \in V_e$

And $h(-x) = \frac{f(-x) - f(x)}{2} = -h(x) \quad \forall x \in \mathbb{R}$

$\therefore h(x) \in V_o$

$\therefore f(x) \in V_e + V_o$

$\therefore V \subseteq V_e + V_o \quad \text{—— (2)}$

From (1) and (2),

$V_e + V_o = V$

(iii) Let $f(x) \in V_e$ and $f(x) \in V_o$

For $\forall x \in \mathbb{R}$, we have

$f(-x) = f(x) \quad \{\because f \in V_e\}$

And $f(-x) = -f(x) \quad \{\because f \in V_o\}$

$\therefore f(x) = -f(x)$

$\Rightarrow f(x) = 0 \quad \forall x \in \mathbb{R}$

$\therefore f(x) \in V_e \cap V_o \Rightarrow f(x) = 0 \quad \forall x \in \mathbb{R}$

$\Rightarrow V_e \cap V_o = \{0\}.$

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Next IAS ID: QAC24MTTN004, MATHS001, 08-07-2024 04:04 AM

Candidates must not write on this margin

Q.2 (b) Show that $\frac{\tan x}{x} > \frac{x}{\sin x}$ for $0 < x < \frac{\pi}{2}$. (15)

For $x=0$, we have

$$\lim_{x \to 0^+} \frac{\tan x}{x} = 1$$

And $$\lim_{x \to 0^+} \frac{x}{\sin x} = 1$$

To show : $\frac{\tan x}{x} > \frac{x}{\sin x}$

$\Rightarrow \frac{\sin^2 x}{\cos x} - x^2 > 0$

$\Rightarrow \sin^2 x - x^2 \cos x > 0$ for $0 < x < \frac{\pi}{2}$

Let $f(x) = \sin^2 x - x^2 \cos x \quad \forall x \in \mathbb{R}$

$f(0) = 0$

$f'(x) = 2 \sin x \cos x + x^2 \sin x - 2x \cos x$

$f'(0) = 0$

Now, for $f'(x) > 0$, we need

$g(x) = 2 \sin x - 2x + x^2 \tan x \geq 0$

$g(0) = 0$

{ dividing $f'(x)$ by $\cos x$ }

$\Rightarrow g'(x) = 2 \cos x - 2 + 2x \tan x + x^2 \sec^2 x$

Candidates must not write on this margin

$g'(0) = 0$

And $g''(x) = -2\sin x + 2\tan x + 2x\sec^2 x + 2x\sec^2 x + x^2 \cdot 2\sec^2 x \tan x$

Now, $\tan x > \sin x \quad \forall x \in \left(0, \frac{\pi}{2}\right)$

$\therefore g''(x) > 0 \quad \forall x \in \left(0, \frac{\pi}{2}\right)$

Also, $g'(0) = 0$

$\therefore g'(x)$ is increasing on $\left(0, \frac{\pi}{2}\right)$

$\therefore g'(x) > 0 \quad \forall x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow g(x)$ is increasing on $\left(0, \frac{\pi}{2}\right)$.

Also, $g(0) = 0$

$\Rightarrow g(x) > 0 \quad \forall x \in \left(0, \frac{\pi}{2}\right)$

$\therefore f'(x) > 0 \quad \forall x \in \left(0, \frac{\pi}{2}\right)$

$\therefore f(x)$ is increasing on $\left(0, \frac{\pi}{2}\right)$

$\therefore f(x) > 0 \quad \forall x \in \left(0, \frac{\pi}{2}\right) \quad \{\because f(0) = 0\}$

$\therefore \frac{\tan x}{x} > \frac{x}{\sin x}$ for $0 < x < \frac{\pi}{2}$.

Candidates must not write on this margin

Next IAS ID: CAC24MTEN004_MATHS001_08.07.2024 04:01 AM

Q.2 (c) Find the equation of two straight lines through the point (1, 1, 1) that intersect the line $x-4=2(y-4)=2(z-1)$ at an angle of 60°. (15)

Candidates must not write on this margin

Any general point on the line

$L_1: x-4 = 2(y-4) = 2(z-1)$

$\Rightarrow P \equiv (4+2r, 4+r, 1+r)$

Line joining P and (1, 1, 1) has d.r's $(3+2r, 3+r, r)$

d.r's of L_1 are $(2, 1, 1)$

For angle of 60°

$$\cos 60^\circ = \frac{2(3+2r)+1(3+r)+1(r)}{\sqrt{2^2+1^2+1^2}\sqrt{(3+2r)^2+(3+r)^2+r^2}}$$

$$\Rightarrow \frac{1}{2} = \frac{9+6r}{\sqrt{6}\sqrt{6r^2+18r+18}}$$

$$\Rightarrow 2(3r^2+3r+3) = 3(3+2r)^2$$

$$\Rightarrow 3r^2+9r+6=0$$

$$\Rightarrow r^2+3r+2=0$$

Next IAS ID: GAC24MTTN004_MATHSS01_08_ST_2024_04_01_AV
Candidates must not write on this margin

$$\Rightarrow r = \frac{-3 \pm \sqrt{9-8}}{2} = -2, -1$$

For $r = -2$:

$P \equiv (0, 2, -1)$

$\therefore$ Required line is

$$\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z-1}{2}$$

For $r = -1$:

$P \equiv (2, 3, 0)$

$\therefore$ Required line is

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{-1}$$

$\therefore$ above two are the required straight lines.

Candidates must not write on this margin

Candidates must not write on this margin

Q.3 (a) Let T be a linear operator on R^3 defined by

$T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$.

Prove that T is invertible and find the formula for T^{-1}. **(15)**

Next IAS ID: QAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Next IAS ID: QAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Next IAS ID: QAC24MTTN004_MATHSS01_08-07-2024 04:06 AM

Candidates must not write on this margin

Q.3 (b) Let $g(x)=f(x)+f(1-x)$ and $f''(x)>0, \forall\ x\epsilon(0,1)$. Find the interval of increase and decrease of $g(x)$. **(15)**

Next IAS ID: 240207TEN006, MATH2024108CE-2024-24161-1

Candidates must not write on this margin

Next IAS ID: QAC24/TEN004/MATHS001_08_07_2024_04_01_AM

Candidates must not write on this margin



Candidates must not write on this margin

Q.3 (c) Find the equation of the cone with vertex at $(2a, b, c)$ and passing through the curve $x^2 + y^2 = 4a^2$ and $z = 0$. Find b and c if the cone also passes through the curve $y^2 = 4a(z + a), x = 0$. Also show that the cone is cut by the plane $y = 0$ in two straight lines and the angle θ between them is given by $\tan\theta = 2$. **(20)**

Next IAS ID: OAS24MTS1004 | ATM2024_08_07_2024_10_421_01

Candidates must not write on this margin

Next IAS | ID: IASCSE0N004 | IP: 103.51.85.8 | 05-07-2024 04:52 PM

Candidates must not write on this margin

Next IAS ID: QAC24MTEN006 MATHS 2024 TEST SERIES-1

Candidates must not write on this margin

Candidates must not write on this margin

Q.4 (a) Let $A=\begin{bmatrix}1 & 3 & 1\\ 2 & 5 & 4\\ 1 & -2 & 2\end{bmatrix}$. Find the matrix B that represents the linear operator A relative to the basis $S(u_1,u_2,u_3)=\{(1,1,0),(0,1,1),(1,2,2)\}$. **(15)**

Let T be the linear transformation for the given operator matrix A.

$\therefore T(1,1,0) = A(1,1,0)^T$

$T(0,1,1) = A(0,1,1)^T$

$T(1,2,2) = A(1,2,2)^T$

Now, $A\,[u_1^T\ u_2^T\ u_3^T]$

$$= \begin{bmatrix}1 & 3 & 1\\ 2 & 5 & 4\\ 1 & -2 & 2\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 2\\ 0 & 1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}4 & 4 & 9\\ 7 & 9 & 20\\ -1 & 0 & 1\end{bmatrix} = [Au_1^T\ Au_2^T\ Au_3^T]$$

Now, let $(\alpha,\beta,\gamma) = a(1,1,0)+b(0,1,1)+c(1,2,2)$

$\Rightarrow a+c=\alpha,\ a+b+2c=\beta,\ b+2c=\gamma$

Candidates must not write on this margin

$\Rightarrow b+c = \beta-\alpha$

$b+2c = \gamma$

$\Rightarrow c = \alpha-\beta+\gamma$

$\therefore b = -2\alpha+2\beta-\gamma$

$\therefore a = \beta-\gamma$

$\therefore (4,7,-1) = 8(1,1,0) + 7(0,1,1) - 4(1,2,2)$

$(4,9,0) = 9(1,1,0) + 10(0,1,1) - 5(1,2,2)$

And $(9,20,1) = 19(1,1,0) + 21(0,1,1) - 10(1,2,2)$

$\therefore$ The required matrix is

$$B = \begin{bmatrix} 8 & 9 & 19 \\ 7 & 10 & 21 \\ -4 & -5 & -10 \end{bmatrix}.$$

Next IAS ID: OAS24/TEN004/MATH0001 08-07-2024 09:42:14

Candidates must not write on this margin

Next IAS ID: OAC24MTT1004 | ATH0001_09_07_2024_0101_1

Candidates must not write on this margin

Q.4 (b) Does the integral $\int_{-1}^{1}\sqrt{\frac{1+x}{1-x}}$ exist? if so, find its value. (15)

The only point of discontinuity in the given integral is at $x=1$

Let $f(x) = \sqrt{\frac{1+x}{1-x}}$

and $g(x) = \frac{1}{\sqrt{1-x}}$

$\lim_{x\to 1^-} \frac{f(x)}{g(x)} = \lim_{x\to 1^-} \sqrt{1+x} = \sqrt{2}$ ($\neq 0$ and finite)

$\therefore \int_{-1}^{1} f$ & $\int_{-1}^{1} g$ converge and diverge together

{ limit comparison test }

$\therefore \int_{-1}^{1} f(x)\,dx$ converges. Let $I = \int_{-1}^{1} f(x)\,dx$

Now, let $\frac{1+x}{1-x} = t^2$

$\Rightarrow \frac{2}{(1-x)^2}\,dx = 2t\,dt$

$\Rightarrow dx = \frac{t\,dt \cdot 4}{(1+t^2)^2}$

Also, $1+x = t^2 - t^2x$

$\Rightarrow 1-x = \frac{2}{1+t^2}$

Next IAS ID: OAC24MTTN004_MATHS001_08_07_2024_0401_AM

Candidates must not write on this margin

$$\therefore I = \int_0^\infty \frac{t \cdot 4t\, dt}{(1+t^2)^2}$$

$$= 4\left\{ \int_0^\infty \frac{(1+t^2)-1}{(1+t^2)^2}\, dt \right\}$$

$$\Rightarrow I = 4\left\{ \underbrace{\int_0^\infty \frac{dt}{1+t^2}}_{I_1} - \underbrace{\int_0^\infty \frac{dt}{(1+t^2)^2}}_{I_2} \right\} \quad \text{— (1)}$$

$$I_1 = \left[\tan^{-1} t\right]_0^\infty = \frac{\pi}{2}$$

For I_2, let $t = \tan\theta$

$$\Rightarrow dt = \sec^2\theta\, d\theta$$

$$\therefore I_2 = \int_0^{\pi/2} \frac{\sec^2\theta\, d\theta}{\sec^4\theta}$$

$$= \int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= \frac{\pi}{4}$$

$$\therefore I = 4\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \boxed{\pi}$$

Next IAS ID: OAOOATTEN004_MATHS001_09-07-2024_12:32:24

Candidates must not write on this margin

Candidates must not write on this margin

Q.4 (c) A point P moves on the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, which is fixed. The plane through P and perpendicular to OP meets the axes in A, B, C respectively. The planes through $A, B\ C$ parallel to yz, zx and xy planes respectively intersect at Q. Prove that the locus of Q is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}$. **(20)**

Let (α, β, γ) be the point Q.

$\therefore$ $A \equiv (\alpha, 0, 0)$

$B \equiv (0, \beta, 0)$

$C \equiv (0, 0, \gamma)$

$\therefore$ plane ABC is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$ — (1)

Now, let coordinates of point P be

$P \equiv (l, m, n)$

$\therefore$ Plane through P and perpendicular to OP is $lx + my + nz = l^2+m^2+n^2$ — (2)

$\therefore$ this plane meets axes at A, B, C

$\therefore$ planes (1) and (2) are the same

$\Rightarrow \frac{l}{1/\alpha} = \frac{m}{1/\beta} = \frac{n}{1/\gamma} = l^2+m^2+n^2$

$\Rightarrow \alpha l = \beta m = \gamma n = l^2+m^2+n^2$

$\Rightarrow \alpha l = l^2 + \left(\frac{\alpha l}{\beta}\right)^2 + \left(\frac{\alpha l}{\gamma}\right)^2$

$\Rightarrow \alpha = \alpha^2 l \left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)$

Candidates must not write on this margin

$$\Rightarrow l = \frac{1}{\alpha\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)}$$

$$\Rightarrow m = \frac{1}{\beta\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)}$$

And $$n = \frac{1}{\gamma\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)}$$

Also, point P i.e. (l, m, n) lies

on $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$$\Rightarrow \frac{l}{a}+\frac{m}{b}+\frac{n}{c}=1$$

$$\Rightarrow \frac{1}{a\alpha\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)} + \frac{1}{b\beta\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)}$$

$$+ \frac{1}{c\gamma\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)} = 1$$

$$\Rightarrow \left(\frac{1}{a\alpha}+\frac{1}{b\beta}+\frac{1}{c\gamma}\right) = \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$$

Next IAS ID: GAS24TEN004 MATHEMATICS CSE TEST SERIES 2024

Candidates must not write on this margin

$\therefore$ locus of Q is (α, β, γ) is

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz} \quad //.$$

Next IAS Test Series 2024 (Mains) 08-07-2024 04:01 AM

Candidates must not write on this margin

Next IAS ID: QAC24MTTN004_MATHS001_08-07-2024 04:02 AM
Candidates must not write on this margin

SECTION-B

Q.5 (a) The eigenvalues of a real symmetric matrix A are -1, 1 and -2. The corresponding eigenvectors are $\frac{1}{\sqrt{2}}(-1\ 1\ 0)^T$, $(0\ 0\ 1)^T$ and $\frac{1}{\sqrt{2}}(-1\ -1\ 0)^T$ respectively. Find the matrix A^4. **(10)**

Consider the matrix of eigenvectors

$$P = \begin{bmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix}$$ → this is an orthogonal matrix.

$\Rightarrow P^T = P^{-1}$.

Now, $P^{-1}AP = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = D$ (say)

$\Rightarrow A = PDP^{-1}$

$\Rightarrow A^2 = (PDP^{-1})(PDP^{-1})$

$= PD(P^{-1}P)DP^{-1}$

$= PD^2P^{-1}$

$\Rightarrow A^4 = PD^4P^{-1}$

$$\Rightarrow A^4 = \begin{bmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 16 \end{bmatrix} P^T$$

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM
Candidates must not write on this margin

$$\Rightarrow A^4 = \begin{bmatrix} -1/\sqrt{2} & 0 & -8\sqrt{2} \\ 1/\sqrt{2} & 0 & -8\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ -1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 17/2 & 15/2 & 0 \\ 15/2 & 17/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ //.

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Q.5 (b) Show that $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)}dx = \frac{\pi}{2}\log(1+a), a \geq 0$ **(10)**

Let $f(a) = \int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)}dx$ for $a \geq 0$

$\Rightarrow f'(a) = \int_0^\infty \frac{x}{x(1+x^2)(1+a^2x^2)}dx$

{Leibnitz rule}

$\Rightarrow f'(a) = \int_0^\infty \frac{dx}{(1+x^2)(1+a^2x^2)}$

$= \frac{1}{(1-a^2)}\left\{\int_0^\infty \frac{dx}{1+x^2} - a^2\int_0^\infty \frac{dx}{1+a^2x^2}\right\}$

$= \frac{1}{(1-a^2)}\left\{\left[\tan^{-1}x\right]_0^\infty - \frac{a^2}{a}\left[\tan^{-1}ax\right]_0^\infty\right\}$

$= \frac{1}{(1-a^2)}\left\{\frac{\pi}{2}(1-a)\right\}$

$\Rightarrow f'(a) = \frac{\pi}{2(1+a)}$

$\Rightarrow f(a) = \int \frac{\pi}{2(1+a)}da$

$\Rightarrow f(a) = \frac{\pi}{2}\log(1+a) + C$

Next IAS ID: OAC24MTTN004_MATHS001_09-07-2024 11:01 AM

Ca
must no
on this mar

But $f(0) = \int_0^\infty 0\,dx = 0$

$\Rightarrow c = 0$

$\therefore f(a) = \int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)}\,dx = \frac{\pi}{2}\log(1+a)$.

Candidates must not write on this margin

Q.5 (c) Prove that the function $f(x,y)=\sqrt{|xy|}$ is not differentiable at the point (0,0), but f_x and f_y both exist at the origin and have the value 0. **(10)**

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{0-0}{h} = 0$$

$$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k)-f(0,0)}{k}$$

$$= \lim_{k\to 0} \frac{0-0}{k} = 0.$$

$\therefore f_x(0,0) = f_y(0,0) = 0.$

Now, let $f(x,y)$ is differentiable at $(0,0)$

$\therefore f(h,k) - f(0,0) = Ah + Bk + \sqrt{h^2+k^2}\,\phi(h,k)$

holds in a neighbourhood of $(0,0)$

$\Rightarrow \sqrt{|hk|} = \sqrt{h^2+k^2}\,\phi(h,k)$ $\quad \{\because A=B=0$ as $f_x=f_y=0\}$

where $\lim_{(h,k)\to(0,0)} \phi(h,k) = 0$

$\Rightarrow \phi(h,k) = \dfrac{\sqrt{|hk|}}{\sqrt{h^2+k^2}}$

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Approaching $(0,0)$ along the curve $h=k$, we get

$$\lim_{\substack{(h,k)\to(0,0)\\(h=k)}} \phi(h,k) = \lim_{h\to 0} \frac{\sqrt{h^2}}{\sqrt{2h^2}}$$

$$= \frac{1}{\sqrt{2}} \neq 0$$

$$\therefore \lim_{(h,k)\to(0,0)} \phi(h,k) \neq 0$$

$\therefore$ contradiction.

$\therefore$ $f(x,y)$ is not differentiable at $(0,0)$.

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Q.5 (d) Prove that the equations of the chord through the point (1, 2, 3) which is bisected by the diametral plane $10x - 24y = 21$ of the paraboloid $5x^2 - 6y^2 = 7z$ are $(x-1)=\frac{1}{2}(y-2)=\frac{1}{3}(z-3)$. (10)

Let us find out the diametral plane that bisects chords with direction ratios (l, m, n) to the given paraboloid.

Let (α, β, γ) be the mid point of chord with d.r's (l, m, n)

$\therefore$ Equation of chord is

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

$\therefore$ A general point on the line is

$(\alpha + lr, \beta + mr, \gamma + nr)$

Putting this in equation of paraboloid

$$5(\alpha+lr)^2 - 6(\beta+mr)^2 = 7(\gamma+nr)$$

Above quadratic in r should have sum of roots $= 0$ { $\because (\alpha, \beta, \gamma)$ is mid point of chord }

$\therefore 10\alpha l - 12\beta m - 7n = 0$

Next IAS ID: OAC24MTTN004_MATHS001_08-07-2024 04:01 AM
Candidates must not write on this margin

$\therefore$ locus of mid points (α, β, γ) is given by $10lx - 12my - 7nz = 0$

↓

this is diametral plane conjugate to d.r.'s (l, m, n)

In given case, plane is $10x - 24y = 21$

$$\Rightarrow \frac{10l}{10} = \frac{-12m}{-24} = \frac{7n}{21}$$

$$\Rightarrow \frac{l}{1} = \frac{m}{2} = \frac{n}{3}$$

$\therefore$ given plane bisects chords with d.r.'s $(1, 2, 3)$

$\therefore$ Chord through the point $P = (1, 2, 3)$ is

$$\boxed{\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}}$$

Next IAS ID: OAC24MTTN004_MATHS001_08-07-2024 04:01 AM
Candidates must not write on this margin

Q.5 (e) Find the equation of the tangent planes to the hyperboloid $2x^2 - 6y^2 + 3z^2 = 5$ which pass through the line $x + 9y - 3z = 0 = 3x - 3y + 6z - 5$. **(10)**

Let $(x + 9y - 3z) + \lambda(3x - 3y + 6z - 5) = 0$ be tangent to the hyperboloid

$\Rightarrow (1 + 3\lambda)x + (9 - 3\lambda)y + (-3 + 6\lambda)z - 5\lambda = 0$

Hyperboloid is $\frac{2}{5}x^2 - \frac{6}{5}y^2 + \frac{3}{5}z^2 = 1$

$\therefore \frac{(1+3\lambda)^2}{2/5} + \frac{(9-3\lambda)^2}{-6/5} + \frac{(-3+6\lambda)^2}{3/5} = (5\lambda)^2$

$\Rightarrow 5\left\{\frac{9\lambda^2 + 1 + 6\lambda}{2} + \frac{9\lambda^2 + 81 - 54\lambda}{-6} + \frac{36\lambda^2 + 9 - 36\lambda}{3}\right\} = 25\lambda^2$

$\Rightarrow 90\lambda^2 - 60 = 30\lambda^2$

$\Rightarrow \lambda = \pm 1$

For $\lambda = 1$:

The tangent plane is

$$\boxed{4x + 6y + 3z - 5 = 0}$$

Candidates must not write on this margin

for $\lambda = -1$:

The tangent plane is

$$2x - 12y + 9z - 5 = 0$$

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Q.6 (a) When a matrix A is said to be similar to another matrix B? Prove that

i. If A is similar to B, then B is similar to A.

ii. Two similar matrices have the same eigenvalues.

Further by choosing appropriately the matrices A and B, show that the converse of (b) above may not be true. **(15)**

If A and B are two square matrices of the same order $n \times n$, A is said to be similar to B if $\exists$ an invertible matrix P s.t. $P^{-1}AP = B$ or $AP = PB$.

(i) If A is similar to B

$\Rightarrow \exists\ P \{|P| \neq 0\}$ s.t. $P^{-1}AP = B$

$\Rightarrow AP = PB$

$\Rightarrow A = PBP^{-1}$

where P^{-1} is invertible.

$\therefore$ B is similar to A.

(ii) Let A and B be similar matrices

$\Rightarrow P^{-1}AP = B$ for some $|P| \neq 0$

Characteristic equation of A is

$|A - \lambda I| = 0$ — (1)

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

for eigenvalues of B, characteristic equation of B is

$$|B - \lambda I| = 0$$

$$\Rightarrow |P^{-1}AP - \lambda I| = 0$$

$$\Rightarrow |P^{-1}AP - \lambda P^{-1}P| = 0$$

$$\Rightarrow |P^{-1}(A - \lambda I)P| = 0$$

$$\Rightarrow |P^{-1}||A - \lambda I||P| = 0 \quad \{\because |MN| = |M||N| \text{ for sq. matrices } M, N\}$$

$$\Rightarrow |A - \lambda I| = 0 \quad \{\because |P^{-1}| = \frac{1}{|P|} \text{ for } |P| \neq 0\}$$

$\therefore$ characteristic equations of A and B are same

$\therefore$ their eigenvalues are same

Converse of (b) may not be true

Consider $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Q.6 (b) If $u = ax^2 + by^2 + cz^2$ where $x^2 + y^2 + z^2 = 1$ and $lx + my + nz = 0$, prove that the stationary values of 'u' satisfy the equation $\frac{l^2}{a-u} + \frac{m^2}{b-u} + \frac{n^2}{c-u} = 0$. (20)

The Lagrange's auxiliary function is

$$F(x, y, z, \lambda_1, \lambda_2) = (ax^2 + by^2 + cz^2) + \lambda_1(x^2 + y^2 + z^2 - 1) + \lambda_2(lx + my + nz)$$

where λ_1, λ_2 are Lagrange's multipliers.

Now, for stationary points,

$$F_x = 0 \Rightarrow 2ax + 2\lambda_1 x + \lambda_2 l = 0 \quad - (1)$$

$$F_y = 0 \Rightarrow 2by + 2\lambda_1 y + \lambda_2 m = 0 \quad - (2)$$

$$F_z = 0 \Rightarrow 2cz + 2\lambda_1 z + \lambda_2 n = 0 \quad - (3)$$

$x \cdot (1) + y \cdot (2) + z \cdot (3)$ gives

$$2u + 2\lambda_1 + \lambda_2(0) = 0$$

$$\Rightarrow \boxed{u = -\lambda_1} \quad - (4)$$

From (1), $2x(a + \lambda_1) = -\lambda_2 l$

$$\Rightarrow x = \frac{-\lambda_2 l}{a + \lambda_1}$$

Similarly, $y = \frac{-\lambda_2 m}{b + \lambda_1}$

Next IAS ID: QAC24MTTN004_MATHS001_08-07-2024 04:01AM
Candidates must not write on this margin

and $z = \dfrac{-\lambda_2 n}{c+\lambda_1}$

Now, $lx + my + nz = 0$

$$\Rightarrow \frac{-\lambda_2 l^2}{a+\lambda_1} + \frac{-\lambda_2 m^2}{b+\lambda_1} + \frac{-\lambda_2 n^2}{c+\lambda_1} = 0$$

$$\Rightarrow \boxed{\frac{l^2}{a-u} + \frac{m^2}{b-u} + \frac{n^2}{c-u} = 0}$$

$\{\because u = -\lambda_1$ from (4)$\}$.

Candidates must not write on this margin

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Q.6 (c) Find the locus of equal conjugate diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (15)

Let (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) be end points of 3 equal conjugate diameters.

$\therefore x_1^2+y_1^2+z_1^2 = x_2^2+y_2^2+z_2^2 = x_3^2+y_3^2+z_3^2$

Also, $(x_1^2+x_2^2+x_3^2)+(y_1^2+y_2^2+y_3^2)+(z_1^2+z_2^2+z_3^2)$

$= a^2+b^2+c^2$

{for conjugate diameters $\sum x_1^2 = a^2$; $\sum y_1^2 = b^2$; $\sum z_1^2 = c^2$}

$\therefore x_1^2+y_1^2+z_1^2 = x_2^2+y_2^2+z_2^2 = x_3^2+y_3^2+z_3^2$

$= \frac{a^2+b^2+c^2}{3}$

$\therefore$ End points of the conjugate diameters satisfy $x^2+y^2+z^2 = \frac{a^2+b^2+c^2}{3}$ — (1)

Also, $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ — (2) satisfied.

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM
Candidates must not write on this margin

$\therefore$ Any such diameter is a line joining $(0,0,0)$ to a curve satisfying ① and ②.

$\therefore$ locus is cone with $(0,0,0)$ as vertex and guiding curve given by ① & ② together.

$\therefore$ required locus is

$$(x^2+y^2+z^2) = \left(\frac{a^2+b^2+c^2}{3}\right)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)$$

$$\Rightarrow \left(\frac{b^2+c^2-2a^2}{a^2}\right)x^2 + \left(\frac{c^2+a^2-2b^2}{b^2}\right)y^2 + \left(\frac{a^2+b^2-2c^2}{c^2}\right)z^2 = 0 \quad //.$$

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Candidates must not write on this margin

Q.7 (a) If S is a skew-Hermitian matrix, then show that $A=(I+S)(I-S)^{-1}$ is a unitary matrix. Also show that every unitary matrix can be expressed in the above form provided -1 is not an eigen value of A. (15)

Next IAS ID: QAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Candidates must not write on this margin

Candidates must not write on this margin

Q.7 (b) Using the transformation $x+y=u, y=uv$, calculate $\iint \left[xy(1-x-y)\right]^{\frac{1}{2}} dxdy$, integration being taken over by the area of the triangle bounded by the lines $x = 0$, $y = 0$, $x + y = 1$. **(15)**

Candidates must not write on this margin

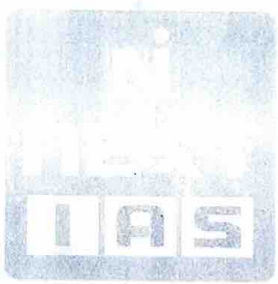
Candidates must not write on this margin

Candidates must not write on this margin

Q.7 (c) Two perpendicular tangent planes to the paraboloid $\frac{x^2}{a}+\frac{y^2}{b}=2z$ intersect in a line lying on the plane $x = 0$. Prove that the line touches the parabola $x=0,\ y^2=(a+b)(2z+a)$. **(20)**

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin



Candidates must not write on this margin

Next IAS ID: QAC24MTT1004, (MAINS)01, 08-07-2024 04:01 AM
Candidates must not write on this margin

Candidates must not write on this margin

Q.8 (a) Let V be complex vector space of 3×3 skew symmetric matrices with complex entries i.e. $V = \{A \in M_{3\times 3}(\mathbb{C}) | A^t = -A\}$. Let $B = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Define a linear transformation $T : V \to V$ by $T(A) = BA - AB$. Find the eigen values and eigenvectors of T. **(15)**

Next IAS_CSE2024_TEN004_MATHS001_08.07.2024 04:01 AM

Candidates must not write on this margin

Next IAS ID: QAC24MTTN004_MATHS001_08-07-2024 04:01:

Candidates must not write on this margin

Q.8 (b) Trace the curve $y^2x^2 = x^2 - a^2$. (20)

Candidates must not write on this margin

Candidates must not write on this margin

Candidates must not write on this margin

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Candidates must not write on this margin

Q.8 (c) Find the length of the normal chord through P of the ellipsoid $\Sigma\frac{x^2}{a^2}=1$ and prove that if it is equal to $4PG_3$, where G_3 is the point where the normal chord through P meets the XY plane, then P lies on the cone $\frac{x^2}{a^6}\left(2c^2-a^2\right)+\frac{y^2}{b^6}\left(2c^2-b^2\right)+\frac{z^2}{c^4}=0.$ **(15)**

Next IAS ID: OAC24MTTN004, MATHS001, 08-07-2024 04:01 AM

Candidates must not write on this margin

Candidates must not write on this margin