

(To be filled by candidate)

Name of Candidate : Kamishak AggarwalRoll No. : MT24MATL1011Contact No. : 9876543210e-mail : 2.comDate of Examination 31/7Exam Centre : Old Rajinder Nagar ☐Online ☒

CSE (MAINS) TEST SERIES-2024

Test-05

MATHEMATICS OPTIONAL

Dated : 21-07-2024

Time Allowed : Three Hours

Maximum Marks: 250

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in Two sections and printed in ENGLISH.

Candidate has to attempt FIVE questions in all.

Question No. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each section.

The number of marks carried by a question/part is indicated against it.

Answers must be written in the medium authorized in the Admission Certificate which must be stated clearly on the cover of this Question-cum- Answer (QCA) booklet in the space provided.

No marks will be given for answers written in a medium other than the authorized one.

Word limit in questions, wherever specified, should be adhered to.

Attempts of question shall be counted in sequential order, unless struck off. Attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question cum Answer (QCA) booklet must be clearly struck off.

DELHI CENTRE:

Old Rajinder Nagar: 27-B, Pusa Road, Metro Pillar no. 115, Near Karol Bagh Metro, New Delhi-110000 | Phone: 8081300200
Mukherjee Nagar: 1422, Main Mukherjee Nagar Road, Near Bata Cinema, New Delhi-110009 | Phone: 8081300200



SECTION-A

1. (a) Show that U_9 is a cyclic group. What are its generators? (10)

$$U_9 = \{1, 2, 4, 5, 7, 8\}$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 7$$

$$2^5 = 5$$

$$2^6 = 1$$

$$\Rightarrow U_9 = \langle 2 \rangle$$

$$\text{or } O(U_9) = 6 \text{ as } O(2) = 6$$

so generators are 2^r where

$$(r, 6) = 1$$

$$\text{i.e. } r = 1, 5$$

i.e.

$$U_9 = \langle 2^1 \rangle, \langle 2^5 \rangle$$

$$= \langle 2 \rangle, \langle 5 \rangle$$

$$5^1 = 5$$

$$5^2 = 7$$

$$5^3 = 8$$

$$5^4 = 4$$

$$5^5 = 2$$

$$5^6 = 1$$

1. (b)

Show that $\mathbb{Z}_7$ is a field. Then, find $([5] + [6])^{-1}$ and $(-[4])^{-1}$ in $\mathbb{Z}_7$.

(10)

$$\mathbb{Z}_7 = \{0, 1, 2, 3, 4, 5, 6\}$$

$+$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

all rows element $\in \mathbb{Z}_7$ (closure)

Associativity by associativity of $\mathbb{Z}$

0 is identity

0 $\in$ every row and column once
ie. unique inverse exist

$(\mathbb{Z}_7, +_7)$ is abelian group

$+$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

$\Rightarrow (\mathbb{Z}_7^{\times}) \times 7$ is abelian.
 $\Rightarrow \mathbb{Z}_7$ is field.

$$[5] + [6] = [4]$$

$$[4]^{-1} = [3]$$

$$-[4] = [3] \quad (\text{additive inverse})$$

$$(-[4])^{-1} = ([3])^{-1} = [5]$$

1. (c)

Discuss the convergence of the sequence $\{x_n\}$ where $x_n = \frac{\sin \frac{n\pi}{2}}{8}$. (10)

$$x_n = \frac{\sin \frac{n\pi}{2}}{8}$$

let $n_1 = 4m_1 + \frac{1}{2}$

$$n_2 = 4m_1 + \frac{3}{2}$$

~~$n_1 = n_2$~~ $n_1 \neq n_2$

$$\lim_{m_1 \rightarrow \infty} n_1 = \lim_{m_1 \rightarrow \infty} n_2$$

~~x_n~~ when $n = n_1$

$$\lim_{n_1 \rightarrow \infty} x_{n_1} = \frac{1}{8} \quad - i$$

x_n when $n = n_2$

$$\lim_{n_2 \rightarrow \infty} x_{n_2} = -\frac{1}{8} \quad - ii$$

from i, ii x_n not converg

A firm produces an alloy having the following specifications:

(10)

- (i) Specific gravity ≤ 0.98
- (ii) Chromium $\geq 8\%$
- (iii) Melting point $\geq 450^\circ\text{C}$

Raw materials A, B and C having the properties shown in the table can be used to make the alloy:

Property	Properties of raw material		
	A	B	C
Specific Gravity	0.92	0.97	1.04
Chromium	7%	13%	16%
Melting point	440°C	490°C	480°C

Costs of the various raw materials per ton are: ₹90 for A, ₹280 for B and ₹40 for C. Formulate the LP model to find the proportion in which A, B and C be used to obtain an alloy of desired properties while the cost of raw materials is minimum.

(10)

let a, b, c be quantity
of A, B, C respectively.

Min $Z: 90a + 280b + 40c$

s.t. $0.92a + 0.97b + 1.04c \leq 0.98$

$0.07a + 0.13b + 0.16c \geq 0.08$

$440a + 490b + 480c \geq 450$

$a, b, c \geq 0.$

1. (c)

Find all possible Taylor's and Laurent's Series expansion of the function

$$f(z) = \frac{2z-3}{z^2-3z+2} \text{ about the point } z=0. \quad (10)$$

$$f(z) = \frac{2z-3}{(z-2)(z-1)} = 2z-3 \left(\frac{1}{z-2} - \frac{1}{z-1} \right)$$

$$= 2 + \frac{1}{z-2} - 2 + \frac{1}{z-1}$$

$$= \frac{1}{z-2} + \frac{1}{z-1}$$

It is ~~convergent~~ ~~as~~ not having any singularity when $|z-0| < 1$ so Taylor possible

$$f(z) = \frac{1}{z-2} + \frac{1}{z-1}$$

$$= f_1(z) + f_2(z)$$

$$f_1(z) = f_1(0) + f_1'(0)z + \frac{f_1''(0)}{2!}z^2 + \dots$$

$$= -\frac{1}{2} + \left(-\frac{1}{4}\right)z + \frac{1}{2}z^2 + \dots$$

$$f_2(z) = -\frac{1}{1} + (-1)z + \frac{z^2}{1^3} + \dots$$

$$f(z) = -\frac{3}{2} - \frac{5}{4}z + z^2\left(\frac{9}{8}\right) + \dots$$

Laurents .

for $|z| < 1$

same as Taylor's .

Next IAS

for $1 < |z| < 2$

$$f(z) = \frac{-1}{2(1 - \frac{z}{2})} + \frac{1}{z(1 - \frac{1}{z})}$$

$$= \frac{-1}{2} \left(1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \dots \right) + \frac{1}{z} \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right)$$

$$= \frac{-1}{2} - \frac{1}{2^3} + \frac{1}{z^3} + \frac{1}{z^2} + \frac{1}{z} - \frac{1}{2} - \frac{z}{4} - \frac{z^2}{8} - \dots$$

for $|z| > 2$

$$f(z) = \frac{+1}{z(1 - \frac{z}{2})} + \frac{1}{z(1 - \frac{1}{z})}$$

$$= \frac{+1}{z} \left[\left(1 + \frac{z}{2} + \frac{4}{z^2} + \frac{8}{z^3} + \dots \right) + 1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right]$$

$$= \frac{1}{z} \left(2 + \frac{3}{z} + \frac{5}{z^3} + \frac{9}{z^3} + \dots \right)$$

2. (a)

If Z is the centre of a group G such that G/Z is cyclic, then show that G is abelian. (15)

Repeated ques $\therefore$ short

$$\frac{G}{Z} = \langle \overline{zg} \rangle \quad g \in G$$

$$\text{let } g_1, g_2 \in G$$

$$\overline{zg}_1, \overline{zg}_2 \in \frac{G}{Z}$$

$$(\overline{zg}_1)(\overline{zg}_2) = (\overline{zg_1g_2}) \quad (\because \overline{zg} \in \frac{G}{Z})$$

$$(\overline{zg_1g_2}) = (\overline{zg_2zg_1}) = (\overline{zg_2g_1}) \quad (\because \overline{zg} \in \frac{G}{Z})$$

$$\begin{aligned} \text{Q. } g_1 &\in \overline{zg}_1 & ; & \quad g_2 \in \overline{zg}_2 \\ g_1 &= z_1g_1 & ; & \quad g_2 = z_2g_2 \\ g_1g_2 &= z_1g_1z_2g_2 \end{aligned}$$

$$= \overline{zg_1g_2} (\overline{zg})^{a_1} (\overline{zg})^{a_2}$$

$$= \overline{zg_1g_2} (\overline{zg})^{a_1+a_2}$$

$$= \overline{zg_2g_1} (\overline{zg})^{a_2+a_1}$$

$$= (\overline{zg})^{a_2} (\overline{zg})^{a_1}$$

$$= z_2g_2z_1g_1$$

$$g_1g_2 = g_2g_1$$

Show that the series $\cos x + \frac{1}{2} \cos 2x + \frac{1}{3} \cos 3x + \dots$ converges uniformly in $(0, 2\pi)$. (15)

$$S_n(x) = \sum f_n(x)$$

$$f_n(x) = \frac{\cos nx}{n}$$

$$f'_n(x) = -\frac{n \sin nx}{n} = -\sin nx$$

$$f'_n(x) = 0$$

$$\Rightarrow nx = 2m\pi$$

$$x = \frac{2m\pi}{n} \quad m \in \mathbb{Z}$$

$$f''_n(x) = -n \cos nx$$

$$\text{at } x = \frac{2m\pi}{n}$$

$$f''_n(x) < 0 \quad \text{if } \cos nx > 0$$

$$\text{for } x \in (0, 2\pi)$$

$\Rightarrow$ Maxima.

$$f_{\max}(x) =$$

$$f_n(x) = \cos nx$$

$$g_n(x) = \frac{1}{n}$$

$$\sum f_n(x) = \left| \frac{\cos\left(\frac{x+nx}{2}\right) \sin \frac{nx}{2}}{\sin \frac{x}{2}} \right| \leq \frac{1}{\sin \frac{x}{2}} < k$$

$$\because \frac{x}{2} \in (0, \pi)$$

i.e. $f_n(x)$ has bounded sum.

$g_n(x) \geq 0$, monotonic and goes to 0.

$\therefore$ By Lebesgue's test $\sum f_n g_n$ converges uniformly

2. (c)

Find the minimum transportation cost using Vogel's approximation method for the following transportation problem:

	Destination				Availability
	D_1	D_2	D_3	D_4	
S_1	9 10	16 0	0 15	5 9	15
Sources S_2	2 0	1 0	3 25	0 5	25
S_3	6 0	4 15	7 0	5 3	20
Demand	10	15	25	10	

(20)

~~44~~ 3, 3, 3 4, 4, 4 2, 2, 2

It is balanced.

~~Initial allotment done~~

u_1	9 0	16 0	15 5	9 10	15	0, 6, 1
u_2	2 10	1 0	3 15	5 0	25	1, 2, 2
u_3	6 0	4 15	7 0	3 0	20	1, 1, 3
	10	15	25	10	60	7, 7
	v_1	v_2	v_3	v_4		

4 3 4 2
12 8 7

$$Z_{\min} = 325$$

Using modi method

$$6 \text{ non zero cell} = 4 + 3 - 1$$

$$\therefore u_1 + v_3 = 15$$

$$u_1 + v_4 = 9$$

$$u_2 + v_1 = 2$$

$$u_2 + v_3 = 3$$

$$u_3 + v_2 = 4$$

$$u_3 + v_3 = 7$$

$$u_1 = 0 \quad v_1 = 14$$

$$u_2 = -12 \quad v_2 = 12$$

$$u_3 = -8 \quad v_3 = 15$$

$$v_4 = 9$$

New allocation -

9	0	16	0	15	10	9
2	10	1	0	3	10	5
6	0	4	15	7	5	3

9	5	16	15	9	10
2	5	1	3	20	5
6		4	15	7	5

$\Delta_{ij} = 0$ for basic

$$u_1 = 0$$

$$u_2 = -7$$

$$u_3 = -3$$

$$v_1 = 9$$

$$v_2 = 7$$

$$v_3 = 10$$

$$v_4 = 4$$

for non basic $\Delta_{34} = 3$ maximum
(3,4) is entering

$$5 + \theta$$

$$5 - \theta$$

$$10 - \theta$$

$$20 + \theta$$

$$5 - \theta$$

$$\theta = 5$$

Candidates
must not write
on this margin

10	16	0	15	9	5
0	1	0	3	25	5
0	4	15	7	0	3

(2,2) cell is basic

$$x_{ij} \leq 0$$

 $\Rightarrow$ optimality reached

$$Z_{\min} = 285$$

$$u_1 + v_1 = 9$$

$$u_1 + v_4 = 9$$

$$u_2 + v_2 = 1$$

$$u_2 + v_3 = 3$$

$$u_3 + v_2 = 4$$

$$u_3 + v_4 = 3$$

$$u_1 = 0, v_1 = 9$$

$$u_2 = -9, v_2 = 10$$

$$u_3 = -6, v_3 = 12$$

$$v_4 = 9$$

3. (a)

Find all the ring homomorphisms from $Z_{20} \rightarrow Z_{30}$.

(15)

$$f: Z_{20} \rightarrow Z_{30}$$

$$f(x) = x \cdot f(1)$$

$$0(f(x)) \mid 30$$

$$\text{Also } f(20) = f(0) = 20f(1) = 0$$

$$\Rightarrow f(1) = 0, 3, 6, 9, 12, 15, 18, 21, 24, 27$$

$$0(f(x)) \mid 30$$

$$0(f(1)) \mid 20$$

$$\text{and } (f(1))^2 = f(1)$$

$$0(f(1)) \mid 30 \text{ and } 0(f(1)) \mid 20 \quad f(1) = a$$

$$\Rightarrow 0(a) = 1, 2, 5, 10.$$

$$\text{Also } 0(a^n) = \frac{0(a)}{(0(a), n)}$$

here a is generator of Z_{30} .

Also Z_{30} cyclic, so no. of element of order a is $\phi(a)$.

$$\cancel{0(a) = 2}$$

$$a = a^n$$

to find n

$$2 = \frac{30}{(0(a), n)}$$

$$\Rightarrow \frac{0(a), n}{n} = 15$$

$$5 = \frac{30}{(o(a), n)} \Rightarrow (o(a), n) = 6$$

$$\Rightarrow n = 6, 12, 18, 24$$

$$10 = \frac{30}{(o(a), n)} \Rightarrow n = 3, 9, 27, 21$$

$\Rightarrow a$ can be
0, 3, 6, 9, 12, 15, 18, 21, 24, 27.

$$1.1 = 1$$

$$f(1.1) = f(1)$$

$$a^2 = a \left(\frac{+}{30} \times 30 \right)$$

$$a = 0, 6, 15, 21$$

so 4 homeomorphisms

If $a_1 > 0$, $b_1 > 0$ and $a_n = \sqrt{a_{n-1}b_{n-1}}$ and $b_n = \frac{2a_{n-1}b_{n-1}}{a_{n-1} + b_{n-1}}$, prove that (i) $\{a_n\}$ and $\{b_n\}$ are monotonic, the one increasing and the other decreasing, (ii) $\{a_n\}$ and $\{b_n\}$ both converge to a common limit. (20)

let $a_2 > a_1$; ~~$b_2 < b_1$~~

~~$a_3 = \sqrt{a_2 b_2}$~~

$a_2 = \sqrt{a_1 b_1} > a_1 = \sqrt{a_1 a_1}$
 $\Rightarrow b_1 > a_1$

~~$a_3 = \sqrt{a_2 b_2}$~~

$b_2 = \frac{2a_1 b_1}{a_1 + b_1}$

$b_2 = \frac{2}{\frac{1}{a_1} + \frac{1}{b_1}}$

$b_1 > a_1$

$\frac{1}{a_1} > \frac{1}{b_1}$

$b_2 < \frac{2}{\frac{1}{b_1} + \frac{1}{b_1}} < b_1$

$b_2 < b_1$

$a_3 = \sqrt{a_2 b_2} > a_2$ ($\because$ GM > HM
 $a_n > b_n$)
 by induction a_n is increasing

~~$a_n > b_n$ ($\because$ GM > HM)~~

$$a_n > b_n \quad (AM > HM)$$

$$a_n = \sqrt{a_{n-1} b_{n-1}} < a_{n-1}$$

$\{a_n\}$ is decreasing.

$$b_n = \frac{2a_{n-1}b_{n-1}}{a_{n-1} + b_{n-1}} > \frac{2b_{n-1}^2}{2b_{n-1}} = b_{n-1}$$

$$b_n > b_{n-1}$$

$\Rightarrow \{b_n\}$ is increasing.

$$\Rightarrow (a_n > b_n) \rightarrow b_1$$

$$\Rightarrow a_n > b_1$$

$$\text{and } b_n < a_1$$

$\Rightarrow$ bounded

hence convergent.

$$b = \frac{2ab}{a+b} ; \quad a = \sqrt{\frac{2ab}{a+b}}$$

$$\Rightarrow a = b$$

$\Rightarrow$ same limit converg

Prove that the function $f(z) = u + iv$, where $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$, $z \neq 0$; $f(0) = 0$ satisfies CR equations at the origin, but the derivative of f at $z = 0$ does not exist. (15)

$$f(z) = \frac{x^3 - y^3}{x^2 + y^2} + i \frac{(x^3 + y^3)}{x^2 + y^2}$$

$u + i v$

u_x at $(0,0)$

$$u_x = \lim_{h \rightarrow 0} \frac{u(h,0) - u(0,0)}{h}$$

$$\lim_{h \rightarrow 0} \frac{h^3}{h} = 0$$

$$u_y = \lim_{h \rightarrow 0} \frac{u(0,h) - u(0,0)}{h}$$

$$= -1$$

$$v_x = \lim_{h \rightarrow 0} \frac{v(h,0) - v(0,0)}{h}$$

$$= 1$$

$$v_y = 1$$

hence CR satisfy.

$$f'(z) \rightarrow \lim_{h \rightarrow 0} \frac{f(z+h) - f(z)}{h}$$

$$\rightarrow \lim_{a+ib \rightarrow 0} \frac{a^3 - b^3 + i(a^3 + b^3)}{(a+ib)(a^2 + b^2)}$$

Next IAS ID: MT24MATL1011, MATHS014, 07-09-2024 04:56 PM

$$b = ma$$

$$\begin{aligned}
 f'(0) &\rightarrow \frac{1-m^3 + i(1+m^3)}{(1+im)(1+m^2)} \\
 &= \frac{[1-m^3 + i(1+m^3)](1-im)}{(1+m^2)^2} \\
 &= \frac{(1-m^3+m+m^4) + i(m^4-m+1+m^3)}{(1+m^2)^2}
 \end{aligned}$$

$$\text{for } m=1$$

$$f' = \frac{2+2i}{2^2}$$

$$\text{for } m=0 \quad f' = \frac{i}{1} = i$$

$\Rightarrow f'(0)$ not unique
hence not diff.

Show that the differential equation of all cones which have their vertex at the origin is $px + qy = z$. Verify that this equation is satisfied by the surface $yz + zx + xy = 0$. (10)

2nd order homogeneous.

$$ax^2 + by^2 + cz^2 + 2hxy + 2fyz + 2gzx$$

$$p = \frac{\partial z}{\partial x}$$

$$2x + 2czp + 2hy + 2fyp + 2gz + 2gxp = 0$$

$$p = \frac{-2gz - 2hy - 2x}{2cy + 2fy + 2gx}$$

$$q = \frac{-2y - 2hx - 2fz}{2cz + 2fy + 2gx}$$

$$px + qy = \frac{-\cancel{ax^2} - 2hxy + 2gzx + \cancel{fyz}}{z}$$

Candidates
must not write
on this margin

$$p = \frac{\partial z}{\partial x}$$

$$yp + px + z + y = 0$$

$$p = -\frac{y+z}{x+y}$$

$$q = -\frac{x+z}{x+y} \quad (4q + z + qx + x = 0)$$

$$px + qy = \frac{-2xy - z(x+y)}{x+y}$$

$$= \frac{-2xy}{x+y} - z$$

$$xy = -z(x+y)$$

$$px + qy = +2z - z = z$$

5. (b)

Multiply the binary numbers $(1111.01)_2$ and $(1101.11)_2$ and check with its decimal equivalent. (10)

$$\begin{array}{r} 1111.01 \\ 1101.11 \\ \hline \end{array}$$

$$\begin{array}{r} 1111 \\ 1101 \\ \hline 1111 \\ 0000 \times \\ 1111 \times \times \\ 1111 \times \times \times \\ \hline 011000011 \end{array}$$

$$\begin{array}{r} 0.01 \\ 0.11 \\ \hline 001 \\ 001 \times \\ \hline 0.0011 \end{array}$$

$$(11000011.0011)_2$$

Verify: =

$$(1111.01)_2 = 15.25$$

$$(1101.11)_2 = 13.75$$

$$= 209.6875$$

5. (c)

Apply Newton Raphson method, to find the real root of transcendental root of the equation $x \log_{10} x = 1.2$, correct to three decimal places. (10)

$$f(x) = x \log_{10} x - 1.2 = \frac{x \ln x}{\log_{10} e} - 1.2$$

$$f'(x) = \frac{\log x}{\log_{10} e}$$

$$f'(x) = \frac{\ln x + 1}{\ln 10}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

~~Let $x_1 = 1$~~

~~$x_{n+1} = x_n - \frac{\frac{x \ln x}{\ln 10} - 1.2}{\frac{\ln x + 1}{\ln 10}}$~~

~~$x_{n+1} = x_n - \frac{x \ln x - 1.2 \ln 10}{\ln x + 1}$~~

~~$x_2 = -1.763$~~

~~$x_3 =$~~

Let $x_1 = 2$

$x_2 = 2.8131$

$x_3 = 2.7411$

$x_4 = 2.7406 = x_{\infty}$

Ans

$$f(x) = x \log_{10} x - 1.2 = \frac{x \ln x}{\log_{10} e} - 1.2$$

$$f'(x) = \frac{\log x}{10}$$

$$f'(x) = \frac{\ln x + 1}{\ln 10}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\text{let } x_1 = 1$$

$$x_{n+1} = x_n - \frac{\frac{x \ln x}{\ln 10} - 1.2}{\frac{\ln x + 1}{\ln 10}}$$

$$x_{n+1} = x_n - \frac{x \ln x - 1.2 \ln 10}{\ln x + 1}$$

$$x_2 = -1.763$$

$$x_3 =$$

$$\text{let } x_1 = 2$$

$$x_2 = 2.8131$$

$$x_3 = 2.7411$$

$$x_4 = 2.7406 = x_5$$

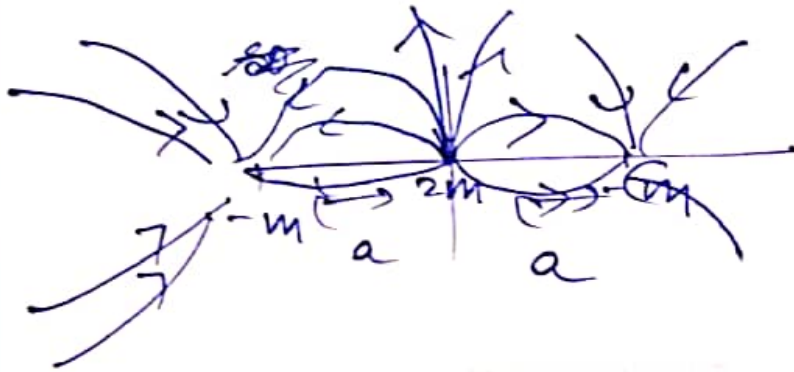
$$\text{Ans } \boxed{2.7406} = \sqrt{2.7411}$$

5. (d)

What arrangement of sources and sinks can have the velocity potential

$$w = \ln\left(z - \frac{a^2}{z}\right)? \text{ Draw the corresponding sketch of the streamlines.} \quad (10)$$

$$w = \ln(z - a) + \ln(z + a) - 2\ln z$$



A rectangular plate swings in a vertical plane about one of its corners. If its period is one second, find the length of its diagonal. (10)



Next IAS ID: MT24MATL1011, MATHS014, 07-09-2024 04:56 PM

Find the surface satisfying $\frac{\partial^2 z}{\partial x^2} = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$. (15)

$$D_x^2 z = 6x + 2$$

homogeneous

$$D_z^2 = 0$$

$$m^2 = 0$$

$$z_c = \phi_1(y) + x \phi_2(y)$$

$$z_p \rightarrow \frac{1}{D^2} (6x + 2)$$

$$= \frac{1}{D} (3x^2 + 2x)$$

$$= x^3 + x^2$$

$$z = \phi_1(y) + x \phi_2(y) + x^2 + x^3$$

touch $z = x^3 + y^3$

along $x + y + 1 = 0$

$$x = t$$

$$y = -t - 1$$

$$3x = 3x^2 = 3t^2$$

$$3y = 3y^2 = 3(t-1)^2 = 3(t+1)^2$$

$$3x = \phi_2(y) + 2x + 3x^2$$

$$\phi_2(-t-1) + 2t + 3t^2 = 3t^2$$

$$\phi_2(-t-1) = -2t$$

$$\phi_2(y) = 2(y+1)$$

$$t-1=y$$

$$t = -y-1$$

$$3y = \phi_1(y) + 2\phi_2(y)$$

$$3(t+1)^2 = \phi_1(y) + 2(2)$$

$$\phi_1(-1-t) = 3t^2 + 3 + 4t$$

$$\phi_1(y) = 3(y+1)^2 + 3 + 4(-y-1)$$

$$\phi_1(y) = 3y^2 + 2 + 2y$$

$$\phi_1(y) = y^3 + 2y + y^2 + C_1$$

equating at $x=t$, $y=-t-1$

$$(-t-1)^3 + 2(-t-1) + (-t-1)^2 + C_1 + 2(2(-t-1))$$

$$+ t^2 + t^3 = t^2 + (-t-1)^3$$

$$-1 + C_1 - 2t^2 + t^3 = 0$$

$$C_1 = 2t^2 + 1 - t^3 = 2(1+y)^2 + 1 - (-1-y)^3$$

$$Z = y^3 + 2y + y^2 + 2(1+y)^2 + 1 - (-1-y)^3 + x(2y+2) + x^2 + x^3$$

(b) A tank in discharging water through an orifice at a depth of x meter below the surface of the water whose area is $A \text{ m}^2$. The following are the values of x for the corresponding values of A .

A	1.257	1.39	1.52	1.65	1.809	1.962	2.123	2.295	2.462	2.650	2.827
x	1.50	1.65	1.80	1.95	2.10	2.25	2.40	2.55	2.70	2.85	3.00

Using the formula $0.0187 = \int_{1.5}^3 \frac{A}{\sqrt{x}} dx$, calculate T the time in seconds for the level of water to drop from 3.0 to 1.5 m above the orifice. (20)

$\frac{A}{\sqrt{x}}$

1.025, 1.081, 1.132, 1.182,

1.249, 1.308, 1.375, 1.438,

1.571, 1.632.

Simpson $1/3$ rule (even interval)

$$= \frac{h}{3} [y_0 + y_{10} + 4(\text{rest}) + 2(y_{\text{even}})]$$

$$= 1.9743$$

$$0.0187 = 1.9743$$

$$T = \underline{110}.$$